9th Grade All Strands
03/12/2008

Student Name: ____________________________

Class: ____________________________

Date: ____________________________

Instructions: Read each question carefully and select the correct answer.

1. The number of mosquitos in Kylie's backyard varies directly as the amount of water in a puddle. When there are 4 gallons of water in the puddle, there are 120 mosquitos in the yard. How many mosquitos would be in the yard if there was 1 gallon of water in the puddle?

A. 116 mosquitos
B. 480 mosquitos
C. 30 mosquitos
D. 481 mosquitos

2. The water temperature of Rocky Creek varies directly as the air temperature in that area. If the water temperature is 61º F when the air temperature is 82º F, what is the water temperature when the air temperature is 76º F? Round your answer to the nearest degree, if necessary.

A. 102º F
B. 66º F
C. 55º F
D. 57º F

3. The cost to have Internet access with Lightening Communications Company is \$9.95 per month plus \$2.50 per hour that a customer is connected to the Internet. What is the minimum number of hours that a customer can access the Internet in a month if her budget allows her to spend more than \$23.50 but less than \$27.50 per month on Internet access? Round your answer to the nearest whole hour that satisfies the inequality.

A. 6 hours
B. 7 hours
C. 5 hours
D. 8 hours

4. The cost to run a sewer pipe from the center of the street to the center of a new house is \$525.00 plus \$12.75 per foot of pipe. What is the maximum number of feet a house can be located from the center of the street if a builder can spend more than \$1,000.00 but less than \$2,000.00 on the sewer pipe installation? Round your answer to the nearest whole foot that satisfies the inequality.

A. 37 feet
B. 116 feet
C. 38 feet
D. 115 feet

5. The time (T) it takes to empty a 5-gallon container of ice cream at a barbecue varies directly with the number of guests (g) requesting ice cream, and indirectly with the number of helpers (h) serving it. The equation that represents this relationship is given by

$$T = \frac{kg}{h}$$ where k is the barbecue constant. If it takes 1.5 hours to empty a container of ice cream when 75 guests request it and there are 3 helpers serving, how long will it take to empty the container if 24 guests request ice cream and 2 helpers are serving?

A. 0.48 hours
B. 0.72 hours
C. 0.95 hours
D. 1.2 hours

6. The volume (V) of a given mass of helium varies directly with its temperature (T), and inversely with its pressure (P). The equation that represents this relationship is given by

$$V = \frac{kT}{P}$$

where k is a helium constant. If a given amount of helium occupies a volume of 189 liters when the temperature is 20º C and the pressure is 121,000 Pascals, what is the volume of the helium when the temperature is 10º C and the pressure is 100,000 Pascals? Round your answer to the nearest tenth.

A. 114.3 liters
B. 121.7 liters
C. 135.2 liters
D. 149.6 liters

7. Add the following matrices.

$$\begin{bmatrix} 1 & -2 \\ 3 & -4 \\ 5 & -6 \end{bmatrix} + \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} =$$

A. $\begin{bmatrix} 8 & -10 \\ 12 & -14 \\ 16 & -18 \end{bmatrix}$

B. $\begin{bmatrix} -1 & 15 \\ -1 & 19 \\ -1 & 23 \end{bmatrix}$

C. $\begin{bmatrix} 14 \\ 18 \\ 22 \end{bmatrix}$

D. $\begin{bmatrix} 8 & 6 \\ 12 & 6 \\ 16 & 6 \end{bmatrix}$

8. Add the following matrices.

$$\begin{bmatrix} 3 & -6 \\ 7 & 4 \end{bmatrix} + \begin{bmatrix} 8 & 1 \\ 2 & -2 \end{bmatrix} =$$

A. $\begin{bmatrix} 10 & 10 \\ -2 & -1 \end{bmatrix}$

B. $\begin{bmatrix} -3 & 9 \\ 10 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 6 \\ 11 \end{bmatrix}$

D. $\begin{bmatrix} 11 & -5 \\ 9 & 2 \end{bmatrix}$

9. Multiply the matrix by the scalar.

$10\begin{bmatrix} -6 & 2 & 0 \end{bmatrix}$

A. $\begin{bmatrix} -40 \end{bmatrix}$

B. $\begin{bmatrix} -60 & 20 & 0 \end{bmatrix}$

C. $\begin{bmatrix} -16 & 12 & 10 \end{bmatrix}$

D. $\begin{bmatrix} 4 & 12 & 10 \end{bmatrix}$

10. Multiply the matrix by the scalar.

$9\begin{bmatrix} 1 & 9 \\ 0 & 3 \\ -3 & -1 \end{bmatrix}$

A. $\begin{bmatrix} 9 & 81 \\ 0 & 27 \\ -27 & -9 \end{bmatrix}$

B. $\begin{bmatrix} 81 \\ 0 \\ 27 \end{bmatrix}$

C. $\begin{bmatrix} 0 & -243 \end{bmatrix}$

D. $\begin{bmatrix} 10 & 18 \\ 9 & 12 \\ -6 & -8 \end{bmatrix}$

11. Simplify.

$32^{\frac{1}{5}}$

A. 2

B. $\frac{1}{160}$

C. $\frac{32}{5}$

D. 160

12. Simplify.

$125^{\frac{2}{3}}$

A. $\frac{250}{3}$

B. 5

C. 25

D. 10

13. Subtract the quantity of three more than half of a number from the quantity of two times the same number decreased by seven.

A. $\frac{5}{2}x - 10$

B. $\frac{3}{2}x - 10$

C. $-\frac{3}{2}x + 10$

D. $-\frac{3}{2}x - 4$

14. Subtract three decreased by six times a number from four times the same number.

A. 2x - 3

B. -10x + 3

C. -2x - 3

D. 10x - 3

15. Evaluate the expression with y = 18.

3 - 2(4y - 5)

A. -131

B. -151

C. 137

D. 157

16. Solve for y.

$4 - (y + 9) = 10$

A. $y = 15$
B. $y = -15$
C. $y = 3$
D. $y = -3$

17. Determine the volume of a sphere with radius = 5 cm. Round your answer to the nearest cubic centimeter. Use $\pi = 3.14$.

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

A. 206 cm^3
B. 65 cm^3
C. 1,047 cm^3
D. 523 cm^3

18. The diameter of a basketball is 9.4 inches. Find its volume. Round your answer to the nearest cubic inch. Use $\pi = 3.14$.

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

A. 435 in^3
B. 388 in^3
C. 7,619 in^3
D. 3,477 in^3

19. Which of the following relations is **not** a function?

A.

$\{(0, -1), (1, 5), (-2, -4), (3, 9), (5, 9)\}$

B.

$\{(0, -1), (1, 5), (-2, -4), (3, 9), (9, 3)\}$

C.

$\{(0, -1), (1, 5), (-2, -4), (3, 9), (3, 5)\}$

D.

$\{(0, -1), (1, 5), (-2, -4), (3, 9), (-3, -9)\}$

20. Which of the following relations is **not** a function?

A.

$\{(-2, -3), (5, -7), (4, 7), (-8, 2), (7, 4)\}$

B.

$\{(-2, -3), (5, -7), (4, 7), (-8, 2), (5, 3)\}$

C.

$\{(-2, -3), (5, -7), (4, 7), (-8, 2), (1, -3)\}$

D.

$\{(-2, -3), (5, -7), (4, 7), (-8, 2), (8, 8)\}$

21. At the Homecoming football game, the home team was raffling off a stereo system. Corey bought 9 raffle tickets and Madeleine bought 7 tickets. There were 600 tickets sold. What is the probability that Corey <u>or</u> Madeleine will win the raffle?

A. $\frac{21}{200}$

B. $\frac{2}{75}$

C. $\frac{7}{40,000}$

D. $\frac{63}{600}$

22. Mark plays baseball on his school team. The probability that he will hit a home run during a game is 0.15. The probability that he will catch a fly ball is 0.74. The probability that the Mark will both hit a home run and catch a fly ball in the same game is 0.11. What is the probability that Mark will either hit a home run <u>or</u> catch a fly ball during a game?

A. 0.11
B. 0.59
C. 0.78
D. 0.89

23. The probability of buying a new battery that does NOT work is 0.002. Of the 6,300 batteries sold this month, how many can be expected to NOT work? Round your answer to the nearest whole number.

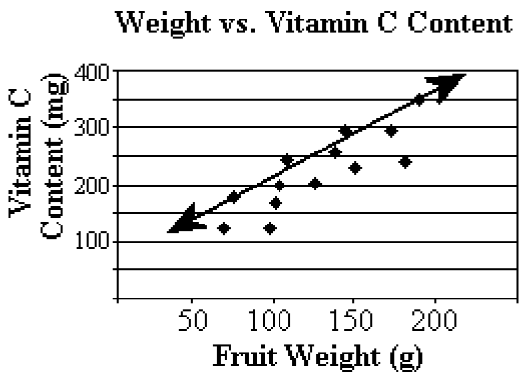
A. 13
B. 6,287
C. 6,288
D. 12

24. In Maria's town, the probability of getting stuck in traffic on the way home from work is 0.14. If she drives home from work 20 times next month, how many times can she expect NOT to be stuck in traffic? Round your answer to the nearest whole number.

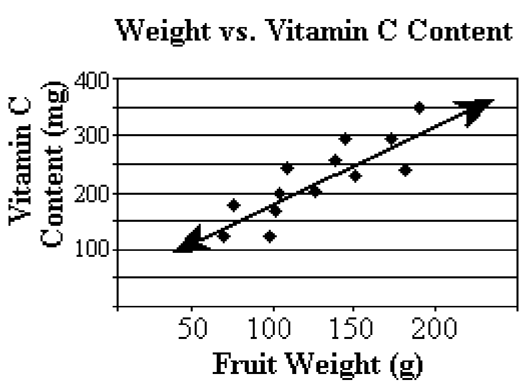
A. 17
B. 18
C. 3
D. 2

25. The graphs below show the weight and Vitamin C content of a variety of fruits. Which graph illustrates the line of best fit for this data?

A.



B.



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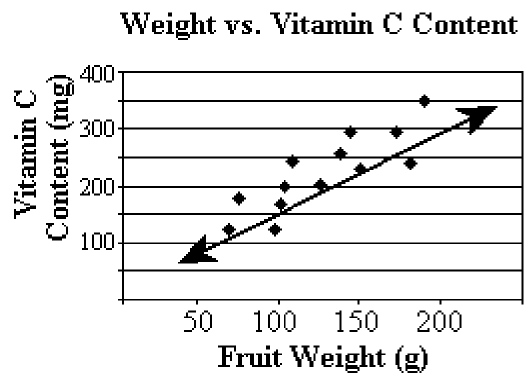
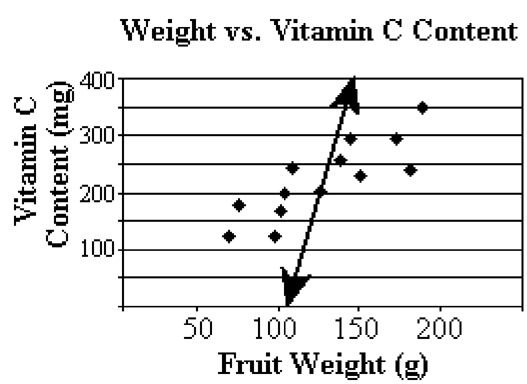
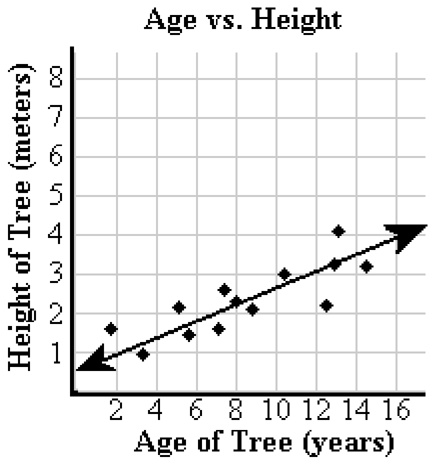


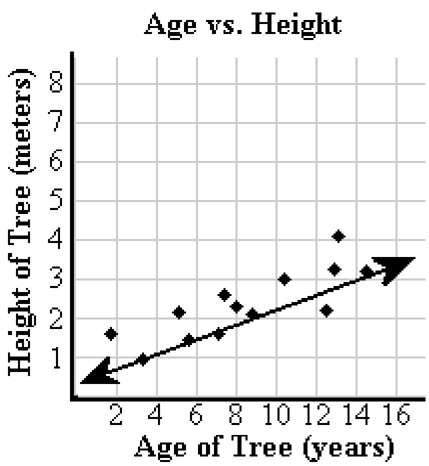
D.



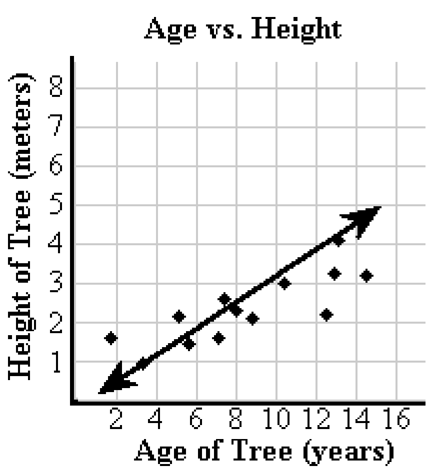
26. The graphs below show the height of several pine trees of different ages. Which graph illustrates the line of best fit for this data?



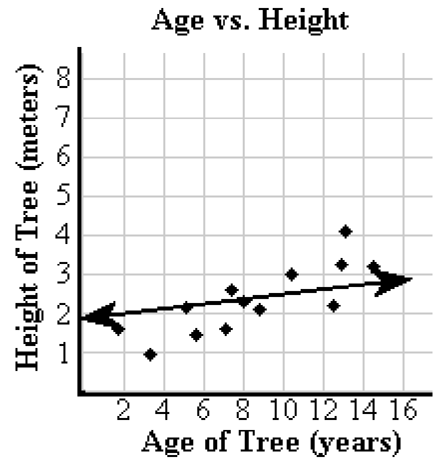
A.



B.



C.



D.

27. Which of the following interest formulas would you use to find interest that is compounded quarterly?

A. $A = Prt$

B. $S = P\left(1 + \frac{r}{n}\right)^{nt}$

C. $S = P\left(\frac{r}{n}\right)^{t}$

D. $A = Pe^{rt}$

28. How much principal must the Ramirez family invest now at 3.5% compounded biannually in order to have enough money to purchase a $20,000.00 pool in 4 years?

A. $17,408.23
B. $17,428.84
C. $11,488.82
D. $22,977.64

29. If you are rolling a six-sided fair die, find the probability of rolling an even number.

A. 1
B. 1/2
C. 1/3
D. 1/6

30. If you are rolling a six-sided fair die, find the probability of NOT rolling a five.

A. 5/6
B. 1/2
C. 1/3
D. 1/6

31. Amy spends twice as much on her house payment as she does on her car payment. Her car payment is 22% of her monthly income of $1,719.76. She spends $80.00 a month on gasoline for her car and $325.00 a month on utility bills. How much money does Amy spend on her house payment?

A. 22%
B. 44%
C. $378.35
D. $756.69

32. Todd has had to make some major repairs on his house this month. They cost him $763.25. This is in addition to his $832.75 mortgage payment and his $132.17 utility bill. If he makes $2,357.68 a month, what percentage of his monthly income went to pay the repair bill? Round your answer to the nearest percent.

A. 35%
B. 6%
C. 32%
D. 73%

33. The number of laps the race cars drive around the track varies inversely with the length of the track. If each car must complete 266 laps around a track that is 1.5 miles long, how many laps would each car need to complete when the track is 1.25 miles long? Round your answer to the nearest whole number, if necessary.

A. 399 laps
B. 319 laps
C. 222 laps
D. 333 laps

34. If p and q vary inversely and $p = 7$ when $q = 10$, find p when $q = 2$.

A. $p = 35$
B. $p = 70$
C. $p = 1.4$
D. $p = 0.7$

35. Solve this system of equations.

$2x - y = 9$
$x + y = 9$

A. x = 4, y = 5
B. x = 9, y = 9
C. x = 18, y = -9
D. x = 6, y = 3

36. A possible step toward solving these equations by addition could be:

$3y - 1 = -2x$
$-9 - 7x = -7y$

A. solving for x in the equation 35x = 20
B. plugging y = 5/7 into the equation 3y - 1 = -2x
C. adding 9 and 1
D. multiplying -1 by the equation 3y - 2x = 1

37. Given the coordinates (1, 9) and (7, 3), find the equation of the line.

A. y = x - 10
B. y = 1/10x
C. y = -10x
D. y = -x + 10

38. A line runs through M(-1, 3) and N(4, -9). Find the equation of the line and solve for y.

A. y = -12/5x - 18 3/5
B. y = -12/5x
C. y = -12/5x - 39/5
D. y = -12/5x + 3/5

39. One number is four less than another number. If three times the smaller number is increased by the larger number, the result is twenty. Find the numbers.

A. x = 8; y = 12
B. x = 6; y = 2
C. x = 4; y = 8
D. x = 2; y = 6

40. The perimeter of a rectangle is one hundred and fifty kilometers. The length of a rectangle is three more than five times the width. Find the length and the width.

A. l = 63; w = 12
B. l = 67.25; w = 15.5
C. l = 70; w = 5
D. l = 104; w = 29

41. Factor completely.

$$6x^2 - 3x - 30$$

A. $3(2x - 5)(x + 2)$
B. $3(2x - 2)(x + 5)$
C. $3(2x + 5)(x - 2)$
D. $3(2x + 2)(x - 5)$

42. If you factor a trinomial into two binomials, how can you manipulate these binomials to come up with the original trinomial?

A. use the quadratic equation
B. multiply the binomials
C. factor completely
D. divide the binomials by the original trinomial

43. Solve the quadratic equation.

$$3x^2 + 8x = 16x - 5$$

A. No real solutions
B. x = -23/3, x = -25/3
C. x = -5/3, x = -1
D. x = 5/3, x = 1

44. Find the discriminant of the quadratic equation and choose the statement which best describes the solution.

$$-2x^2 + 4x + 8 = 0$$

A. The solutions are two distinct real numbers.
B. The solutions are two distinct imaginary numbers (complex conjugates).
C. There are no real solutions.
D. There is only one real solution.

45. Simplify.

$$4x - 8 + 3(x - 4)$$

A. $7x - 12$
B. $7x - 20$
C. $7x + 4$
D. $x + 20$

46. Simplify.

$4a^2 - 10a - 21$

A. $4a^2 - 10a - 21$
B. $3a - 10$
C. $4a^2 - 2a - 27$
D. $4a^2 - 38a - 21$

47. Simplify.

$$\frac{8x^5y^6z}{12x^3y^2z^2}$$

A. $\frac{2xy}{3z}$

B. $\frac{4x^2y^4}{6z}$

C. $\frac{2x^8y^8z^3}{3}$

D. $\frac{2x^2y^4}{3z}$

48. Simplify.

$(2a^2)^3$

A. $2a^6$
B. $8a^6$
C. $2a^5$
D. $6a^6$

49. Multiply and simplify.

$(x^2 + 4x - 7)(3x - 4)$

A. $3x^3 + 16x^2 - 5x + 28$
B. $3x^3 + 8x^2 - 37x + 28$
C. $3x^3 + 8x^2 + 37x - 28$
D. $3x^3 - 16x^2 + 5x - 28$

50. Multiply and simplify.

$(1 - 9x)(x^2 - 5x - 3)$

A. $-9x^3 + 46x^2 + 22x - 3$
B. $-9x^3 - 44x^2 - 32x + 3$
C. $9x^3 - 46x^2 - 22x + 3$
D. $9x^3 + 44x^2 + 32x - 3$

51. Divide $(72x^2 + 18)$ by $(-6x + 5)$.

A. $-12x - 13 - \frac{65}{6x - 5}$

B. $12x + 13 + \frac{65}{6x - 5}$

C. $12x + 10 - \frac{68}{6x + 5}$

D. $-12x - 10 - \frac{68}{6x - 5}$

52. Simplify.

$$\frac{16b^4 + 54b}{2b}$$

A. $14b^4 + 52b$
B. $8b^3 + 27$
C. $8b^4 + 27b$
D. $14b^3 + 52$

53. Simplify and evaluate the expression for x = 9.

6 - 5(3 - 5(2 - 3x))

A. -634
B. 616
C. 141
D. 50

54. Determine the value of the question mark.

$$n = 3$$
$$5\left(\frac{n}{2}\right) = ?$$

A. 30
B. 15
C. 7 1/2
D. 9 3/4

55. Solve for x.

$$\sqrt{4x-2}+3=5$$

A. $x = \frac{3}{2}, \frac{-1}{2}$

B. $x = \frac{3}{2}$

C. $x = \sqrt{2}$

D. $x = \frac{\sqrt{2}+2}{4}$

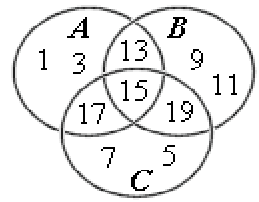
56. The product of a number and eight is decreased by two. The square root of the result is equal to sixteen. Find the number.

A. 32 1/4
B. 31 3/4
C. 3/4
D. 0.7

57. Find the solution.
3(x + 1) = 2x + 10

A. x = 13
B. x = -13
C. x = 7
D. x = -7

58. Given sets *A*, *B*, and *C* expressed in the diagram, which answer choice is <u>FALSE</u>?



A. $B \cap C = \{19\}$

B. $A \cup B = \{1,3,9,11,13,15,17,19\}$

C. $A \cap C = \{15,17\}$

D. $B \cup C = \{5,7,9,11,13,15,17,19\}$

59. Solve the inequality.

$$|3x - 10| > 43$$

A. $x < -11$

B. $x > 17\frac{2}{3}$ or $x < -11$

C. $-11 < x < 17\frac{2}{3}$

D. $x > -11$

60. Solve for *x*.

$$\left|\frac{1}{9}x + \frac{5}{6}\right| = 2\frac{2}{3}$$

A. $x = \frac{33}{2}$

B. $x = \frac{33}{2}$ or $x = -\frac{63}{2}$

C. $x = \frac{63}{2}$ or $x = -\frac{33}{2}$

D. $x = \frac{63}{2}$

9th Grade All Strands
Answer Key
03/12/2008

1.	**C**	Direct Variation
2.	**D**	Direct Variation
3.	**A**	Inequalities - C
4.	**D**	Inequalities - C
5.	**B**	Joint and Combined Variation
6.	**A**	Joint and Combined Variation
7.	**D**	Add Matrices
8.	**D**	Add Matrices
9.	**B**	Scalar Multiplication With Matrices
10.	**A**	Scalar Multiplication With Matrices
11.	**A**	Exponential Notation - F
12.	**C**	Exponential Notation - F
13.	**B**	Polynomials: Subtraction
14.	**D**	Polynomials: Subtraction
15.	**A**	Equations: Order of Operations
16.	**B**	Equations: Order of Operations
17.	**D**	Volume of Spheres
18.	**A**	Volume of Spheres
19.	**C**	Functions/Relations - A
20.	**B**	Functions/Relations - A
21.	**B**	Mutually Exclusive/Inclusive
22.	**C**	Mutually Exclusive/Inclusive
23.	**A**	Theoretical Probability - B
24.	**A**	Theoretical Probability - B
25.	**B**	Estimate Line of Best Fit: Scatter Plot
26.	**A**	Estimate Line of Best Fit: Scatter Plot
27.	**B**	Compound Interest
28.	**A**	Compound Interest
29.	**B**	Theoretical Probability - A
30.	**A**	Theoretical Probability - A
31.	**D**	Budget: Creation/Application
32.	**C**	Budget: Creation/Application
33.	**B**	Inverse Variation
34.	**A**	Inverse Variation
35.	**D**	Equations: Systems
36.	**B**	Equations: Systems
37.	**D**	Equations of a Line
38.	**D**	Equations of a Line
39.	**C**	Number Relation Problems
40.	**A**	Number Relation Problems
41.	**A**	Factoring
42.	**B**	Factoring
43.	**D**	Quadratic Formula
44.	**A**	Quadratic Formula
45.	**B**	Polynomials: Addition
46.	**A**	Polynomials: Addition
47.	**D**	Exponential Notation - E
48.	**B**	Exponential Notation - E
49.	**B**	Polynomials: Multiplication
50.	**A**	Polynomials: Multiplication
51.	**D**	Polynomials: Division
52.	**B**	Polynomials: Division
53.	**A**	Expressions: Evaluating & Simplifying
54.	**C**	Expressions: Evaluating & Simplifying
55.	**B**	Radicals: Equations
56.	**A**	Radicals: Equations
57.	**C**	Sets/Subsets/Solution Sets
58.	**A**	Sets/Subsets/Solution Sets
59.	**B**	Absolute Value: Solve
60.	**B**	Absolute Value: Solve

Study Guide

9th Grade All Strands
03/12/2008

Direct Variation
Variation equations are formulas that show how one quantity changes in relation to one or more other quantities. There are four types of variation: direct, indirect (or inverse), joint, and combined.

Direct variation equations show a relationship between two quantities such that when one quantity increases, the other also increases, and when one quantity decreases, the other also decreases. We can say that *y* varies directly as *x*, or *y* is proportional to *x*. Direct variation formulas are of the form $y = kx$, where the number represented by *k* does not change and is called a constant of variation.

Indirect variation equations are of the form $y = k/x$ and show a relationship between two quantities such that when one quantity increases, the other decreases, and vice versa.

This skill focuses on direct variation. The following is an example of a direct variation problem.

The amount of money in a paycheck, *P*, varies directly as the number of hours, *h*, that are worked. In this case, the constant *k* is the hourly wage, and the formula is written $P = kh$. If the equation is solved for *k*, the resulting equation shows that *P* and *h* are proportional to each other.

$k = \frac{P}{h}$ Therefore, when two variables show a direct variation relationship, they are proportional to each other. Direct variation problems can be solved by setting up a proportion in the form below.

$$\frac{P_1}{h_1} = \frac{P_2}{h_2}$$

P_1 = the amount of the first paych
h_1 = the number of hours worked
P_2 = the amount of the second pa
h_2 = the number of hours worked

Example 1: The amount of fuel needed to run a textile machine varies directly as the number of hours the machine is running. If the machine required 8 gallons of fuel to run for 24 hours, how many gallons of fuel were needed to run the machine for 72 hours? Round your answer to the nearest tenth of a gallon, if necessary.

(1) $\frac{8 \text{ gallons}}{24 \text{ hours}} = \frac{g \text{ gallons}}{72 \text{ hours}}$

(2) $\frac{8 \text{ gallons}}{24 \text{ hours}} \times \frac{g \text{ gallons}}{72 \text{ hours}}$

(3) (g gallons)(24 hours) = (8 gallons)(72 hours

(4) $\frac{(g \text{ gallons})(24 \text{ hours})}{(24 \text{ hours})} = \frac{(8 \text{ gallons})(72 \text{ hours}}{(24 \text{ hours})}$

(5) $\frac{(g \text{ gallons})\cancel{(24 \text{ hours})}}{\cancel{(24 \text{ hours})}} = \frac{(8 \text{ gallons})\overset{3}{\cancel{(72 \text{ hours}}}}{\underset{1}{\cancel{(24 \text{ hours})}}}$

(6) $g = 24$ gallons

Step 1: Set up the proportion. Since the machine used 8 gallons of fuel in 24 hours, the left side of the proportion should be 8 gallons over 24 hours. The number of gallons that the machine used in 72 hours needs to be found, so the right side of the proportion should be *g* gallons over 72 hours.
Step 2: Cross-multiply across the equal sign.
Step 3: Set up the cross-multiplication equation.
Step 4: Divide both sides of the equation by

24 hours to isolate *g* gallons.
Step 5: Reduce the fractions on both sides of the equal sign.
Step 6: Simplify by multiplying the numbers remaining on the right side of the equal sign (8 gallons $\times$ 3).

Answer: 24 gallons

Example 2: The price of jellybeans, *j*, varies directly as the number of pounds, *p*, that are purchased. Find the equation that relates the two variables if jellybeans are \$1.95 per pound.

(1) $y = kx, j = kp$
(2) $j = 1.95p$

Step 1: Remember that the formula for direct variation is: $y = kx$ and substitute the variables from the question into the appropriate places.
Step 2: Since the jellybeans are always \$1.95 per pound, the constant, *k*, equals 1.95. Substitute 1.95 into the equation for *k*.

Answer: $j = 1.95p$

Activities that can help reinforce the concept of direct variation are as follows.

1. Have students solve the equation $y = kx$ for *k*, and then substitute two sets of (x, y) values into the equation and compare the values for *k*. If they are the same, then *x* and *y* have a direct variation relationship.
2. Have the student think of scenarios that show a direct variation relationship. Then, make up numbers to go with the relationships and have the students practice solving them.

Inequalities - C

An inequality is a number sentence that uses "is greater than," "is greater than or equal to," "is less than," "is less than or equal to," or "is not equal to" symbols. For example, $6n > 4$ is a number sentence with an inequality symbol, and is read "six times *n* is greater than four." Inequalities can be identified in real world situations by expressions such as "is less than," "is more than," "at least," and "at most." Sentences can be translated to number sentences using the following symbols.

Symbol	Meaning	Associat Word Phra
>	is more than	more th
<	is less than	less tha
≥	is greater than or equal to	at least
≤	is less than or equal to	at most

An inequality can be solved for a variable (a letter that represents a number) in the same way that an equation is solved. An example of solving an inequality follows.

Example 1: Solve the inequality for *x*.

$$2x + 7 > 11$$

(1)

$$\begin{array}{r} 2x + 7 > 11 \\ -7 \quad -7 \\ \hline 2x > 4 \end{array}$$

(2)

$$\frac{2x}{2} > \frac{4}{2}$$
$$x > 2$$

Step 1: Subtract 7 from each side of the inequality symbol to isolate the variable term, $2x$, and simplify.
Step 2: Divide both sides of the inequality by 2 to completely isolate *x*.

Answer: $x > 2$

A compound inequality has more than one condition and can be identified in a number sentence by two inequality symbols, or in a real world application by the words "and" or "or." For example, $3 < x < 9$ is a compound inequality. To read a compound inequality, start in the center and read left: $x > 3$, then go back to the center and read to the right: $x < 9$. The compound inequality is a combination of $x > 3$ AND $x < 9$. Solving a compound inequality is very similar to solving a single inequality.

Example 2: Solve the compound inequality for x.

$$4 < 3x + 1 < 7$$

(1)

$$\begin{array}{rcl} 4 < & 3x + 1 & < 7 \\ -1 & -1 & -1 \\ \hline 3 < & 3x & < 6 \end{array}$$

(2)

$$\frac{3}{3} < \frac{3x}{3} < \frac{6}{3}$$

$$1 < x < 2$$

Step 1: Subtract 1 from the left, center, and right side of the number sentence to isolate the variable term, $3x$.
Step 2: Divide the left, center, and right side of the number sentence by 3 to isolate x.

Answer: $1 < x < 2$. This answer can be interpreted to mean that x is greater than 1 and x is less than 2.

Although the above examples do not include negative numbers, students must remember to switch the inequality sign when multiplying or dividing by a negative number.
Once the student is familiar with solving compound inequalities, he or she should be ready to solve them in real world situations.

Example 3: The telephone company charges $21.95 per month for basic service plus $0.17 per local call. What is the maximum number of calls that can be made in a month if a family can spend at least $27.50 and at most $32.75 per month for telephone service? Round your answer to the nearest whole number of calls that satisfies the inequality.

(1) $\$27.50 \le \$21.95 + \$0.17c \le \$32.75$

(2)

$$\begin{array}{rcl} \$27.50 \le & \$21.95 + \$0.17c & \le \$32.75 \\ -\$21.95 & -\$21.95 & -\$21.95 \\ \hline \$5.55 \le & \$0.17c & \le \$10.80 \end{array}$$

(3) $\dfrac{\$5.55}{\$0.17} \le \dfrac{\$0.17c}{\$0.17} \le \dfrac{\$10.80}{\$0.17}$

(4) $32.65 \le c \le 63.53$

(5) maximum = 63 calls

Step 1: Translate the information in the problem to a compound inequality. Let c represent the number of calls. The family gets charged $21.95 + 0.17c$ each month (the service charge plus local calls). Since the charges are **at least** $27.50 and **at most** $32.75, place the minimum amount of $27.50 to the left of the expression that represents the charges and the maximum amount of $32.75 to the right of the expression. Refer to the chart above to see which inequality symbol to use. Since the question uses the words *at least* and *at most*, the $\le$ needs to be used. Place the $\le$ on either side of the charges expression.
Step 2: Isolate the variable term 0.17c$ by subtracting $21.95 from the left, center, and right of the compound inequality. Simplify.
Step 3: Divide each term of the new compound inequality by $0.17 to isolate c.
Step 4: Simplify the inequality.
Step 5: Since the maximum number of calls must be less than or equal to 63.53, the whole number that will satisfy the inequality is 63. Rounding up to 64 would no longer satisfy the inequality because 64 is greater than 63.53.

Answer: 63 calls

Note: If the question had asked what the **minimum** number of calls would be, the answer would be 33, since the minimum number of calls has to be greater than or equal to 32.65.

An activity that can help reinforce the concept of inequalities is to ask the student

to make a budget using the amount of money he or she earns each month (approximate or make up an amount if necessary). Then, make up scenarios using the budget. For example:
It costs \$120 per month to insure a car, plus \$2.05 per gallon of gasoline used. If you have budgeted at least \$150.00 and at most \$200.00 for car expenses this month, what is the maximum number of gallons of gas you can buy?

Joint and Combined Variation
Variation equations are formulas that show how one quantity changes in relation to one or more quantities. There are four types of variation: direct, indirect (or inverse), joint, and combined. This skill focuses on joint and combined variation.

Direct variation equations show a relationship between two quantities such that when one quantity increases, the other also increases, and vise versa. We can say that y varies directly as x. Direct variation formulas are of the form $y = kx$, where the number represented by k does not change and is called a constant of variation. For example, the amount of money in a paycheck (P) varies directly as the number of hours (h) worked. In this case, the constant k is the hourly wage, and the formula is written $P = kh$.

Indirect variation formulas show that when one quantity increases, the other quantity decreases, and vise versa. For example, when the price of an item increases, the demand decreases. Indirect variation formulas are of the form $y = k/x$.

Joint variation formulas show the relationship between a quantity and the product of two other quantities. They are of the form $y = kxz$. For example, the equation for the area of a triangle states that the area is equal to one half times base times height, or $A =$? bh. The number ? is the constant of variation, and is always the same for the area of a triangle.

Combined variation formulas show how a quantity varies directly with one or more quantities and indirectly with one or more different quantities. They are of the form $y = (kx)/z$. For example, the sales (S) of a company vary directly as the amount spent on advertising (a), and indirectly as the price (p) of the item sold. The formula is written as $S = (ka)/p$. When the amount spent on advertising increases, sales increase because sales and advertising vary directly. When the price increases, sales decrease, because price and sales vary indirectly.

To solve variation problems, follow these steps.

Step 1: Substitute the quantities given for the letters they represent.
Step 2: Solve for the constant k.
Step 3: Set up the equation again.
Step 4: Substitute given quantities and the value of k that was just determined.
Step 5: Solve for the quantity needed.

Example 1: The surface area (S) of a cube varies jointly with the length (l) and width (w) of one of the faces. The equation that represents this relationship is given by

$$S = klw$$

where k is a constant that needs to be determined. If the surface area of a cube is 150 square inches when the length and width of each face are 5 inches, what are the length and width of each face when the surface area is 216 square inches?

(1) $S = 150,\ l = 5,\ w = 5,\ k = ?$

(2) $150 = k(5)(5)$

$$\frac{150}{25} = \frac{25k}{25}$$

$$k = 6$$

(3) $S = 216,\ k = 6,\ l = ?,\ w = ?$

$$216 = (6)(l)(w)$$

(4) $$\frac{216}{6} = \frac{(6)(l^2)}{6}$$

$$36 = l^2$$

$$\sqrt{36} = \sqrt{l^2}$$

$$6 = l,\ 6 = w$$

Step 1: Write the values for the variables that are known (do not use the values for a surface area of 216 because the constant k is not known).
Step 2: Substitute the values into the formula and solve for k. Multiply the terms on the right side of the equal sign to get $25k$. Isolate the k by dividing each side of the equation by 25. $k = 6$.
Step 3: Now that the constant is known, it is possible to determine the length and width of a cube that has a surface area of 216 in.^{2}. Write the values for the variables that are known for a surface area of 216.
Step 4: Substitute the values into the variation equation and solve for l and w. Multiply the right side of the equation to get $6l^2$. Since length and width are the same for a cube, $(l)(w)$ can be written l^2 or w^2. Divide each side of the equation by 6 to isolate the l^2. To remove the square from l^2, take the square root of each side of the equation. $l = 6$ and $w = 6$ (because l and w are the same).
Answer: length = 6 inches and width = 6 inches

Example 2: The amount of monthly sales (S) for a company that makes a certain type of chair varies directly with the amount spent on advertising each month (a) and indirectly with the price (p) of the chair. The equation that represents this relationship is given by

$$S = \frac{ka}{p}$$

where k is a constant to be determined. If the sales for July were $40,000.00 when advertising costs were $100.00 and the price of the chair was $50.00, what were the sales for August if advertising costs were increased to $135.00 and the price of the chair was decreased to $45.00?

(1) $S = 40{,}000;\ a = 100;\ p = 50$

(2) $$40{,}000 = \frac{k(100)}{50}$$

$$40{,}000 = \frac{100k}{50}$$

(3) $$\frac{40{,}000}{2} = \frac{2k}{2}$$

$$20{,}000 = k$$

(4) $S = ?;\ k = 20{,}000;\ a = 135;\ p = 45$

$$S = \frac{20{,}000(135)}{45}$$

$$S = \frac{2{,}700{,}000}{45}$$

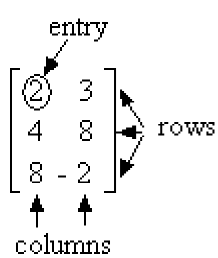
$$S = 60{,}000$$

Step 1: Write the values for the variables that are known (do not use the values for advertising cost of $135 and price of $45 because the constant k is not known).
Step 2: Substitute the values into the formula and solve for k. Multiply the terms on the top of the fraction to get $100k$. Divide $100k$ by 50 to further simplify the right side of the equation and get $2k$. Isolate the k by dividing both sides of the equation by 2, yielding $k = 20{,}000$.
Step 3: Now that the constant is known, it is

possible to determine the sales for advertising costs of \$135 and price of \$45. Write the values for the variables that are known for these constraints.
<u>Step 4</u>: Substitute the values into the variation equation and solve for *S*. Multiply the numbers on the top of the fraction to get 2,7000,000. Divide 2,700,000 by 45 to continue to simplify the equation and determine that the sales were \$60,000.00
Answer: \$60,000.00

An activity that can help reinforce the concept of variation is to describe situations involving joint and combined variation, and then have the student write the equations relating the quantities. For example, "The volume of a cylinder varies jointly as the square of the radius of the cylinder and its height." The student would write: $V = kr^2 h$.

Add Matrices
A <u>matrix</u> is an array of numbers arranged in rows and columns. Rows are horizontal and columns are vertical. An <u>entry</u> is a number in the matrix.



The plural of matrix is "matrices." Two matrices can be added together, if they have the same number of rows and the same number of columns. Addition of matrices is done by adding corresponding entries, or entries that are in the same position within their respective matrix.

A matrix with 2 rows and 3 columns is identified as a 2×3 matrix, and a matrix with 3 rows and 2 columns is a 3×2 matrix. The number of rows is indicated first, and the number of columns is indicated second.

Example 1: Add the following 2×2 matrices.

$$\begin{bmatrix} 3 & -8 \\ 4 & 20 \end{bmatrix} + \begin{bmatrix} 8 & -4 \\ -15 & -7 \end{bmatrix} =$$

(1)

$$\begin{bmatrix} 3 & -8 \\ 4 & 20 \end{bmatrix} + \begin{bmatrix} 8 & -4 \\ -15 & -7 \end{bmatrix}$$

(2)

$$\begin{bmatrix} 3+8 & -8+-4 \\ 4+-15 & 20+-7 \end{bmatrix}$$

(3)

$$\begin{bmatrix} 11 & -12 \\ -11 & 13 \end{bmatrix}$$

<u>Step 1:</u> Rewrite the problem.
<u>Step 2:</u> Add corresponding entries.
<u>Step 3:</u> Simplify.

Answer: $\begin{bmatrix} 11 & -12 \\ -11 & 13 \end{bmatrix}$

Example 2: Add the following matrices.

$$\begin{bmatrix} -12 & 6 & 10 \\ 7 & 3 & -8 \\ -13 & 4 & 20 \end{bmatrix} + \begin{bmatrix} 14 & 3 & 4 \\ 8 & -4 & 5 \\ -15 & -7 & 3 \end{bmatrix} =$$

(1)

$$\begin{bmatrix} -12 & 6 & 10 \\ 7 & 3 & -8 \\ -13 & 4 & 20 \end{bmatrix} + \begin{bmatrix} 14 & 3 & 4 \\ 8 & -4 & 5 \\ -15 & -7 & 3 \end{bmatrix}$$

(2)

$$\begin{bmatrix} -12+14 & 6+3 & 10+4 \\ 7+8 & 3+-4 & -8+5 \\ -13+-15 & 4+-7 & 20+3 \end{bmatrix}$$

(3)

$$\begin{bmatrix} 2 & 9 & 14 \\ 15 & -1 & -3 \\ -28 & -3 & 23 \end{bmatrix}$$

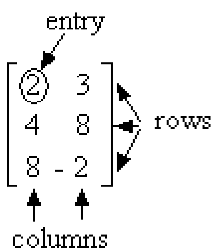
<u>Step 1:</u> Rewrite the problem.
<u>Step 2:</u> Add corresponding entries.
<u>Step 3:</u> Simplify.

Answer: $\begin{bmatrix} 2 & 9 & 14 \\ 15 & -1 & -3 \\ -28 & -3 & 23 \end{bmatrix}$

An activity that can help reinforce the concept of matrices is to ask the value of the entry in a given row and column in a matrix. After the student is able to identify entries, ask him or her to add the corresponding entries.

Scalar Multiplication With Matrices

A matrix is an array of numbers arranged in rows and columns. Rows are horizontal and columns are vertical. A scalar is a real number that can be multiplied by a matrix. In scalar multiplication, each entry is multiplied by the scalar. An entry is a number in the matrix.



Example 1: Multiply the matrix by the scalar.

$$3\begin{bmatrix} 1 & 4 \\ -1 & 9 \end{bmatrix}$$

(1)	(2)	(3)
$3\begin{bmatrix} 1 & 4 \\ -1 & 9 \end{bmatrix}$	$\begin{bmatrix} (3)(1) & (3)(4) \\ (3)(-1) & (3)(9) \end{bmatrix}$	$\begin{bmatrix} 3 & 12 \\ -3 & 27 \end{bmatrix}$

Step 1: Rewrite the problem.
Step 2: Multiply each entry in the matrix by the scalar, 3.
Step 3: Simplify.

Answer: $\begin{bmatrix} 3 & 12 \\ -3 & 27 \end{bmatrix}$

Example 2: Multiply the matrix by the scalar.

$$-2\begin{bmatrix} 11 & 2 \\ -9 & 12 \\ -6 & 3 \end{bmatrix}$$

(1)	(2)	(3
$-2\begin{bmatrix} 11 & 2 \\ -9 & 12 \\ -6 & 3 \end{bmatrix}$	$\begin{bmatrix} (-2)(11) & (-2)(2) \\ (-2)(-9) & (-2)(12) \\ (-2)(-6) & (-2)(3) \end{bmatrix}$	$\begin{bmatrix} -22 \\ 18 \\ 12 \end{bmatrix}$

Step 1: Rewrite the problem.
Step 2: Multiply each entry in the matrix by the scalar, - 2.
Step 3: Simplify.

Answer: $\begin{bmatrix} -22 & -4 \\ 18 & -24 \\ 12 & -6 \end{bmatrix}$

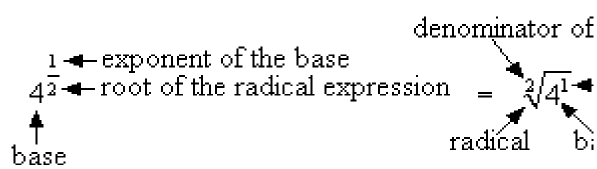
Exponential Notation - F

In the expression 3^2, the number 2 is called an **exponent** and the number 3 is called a **base**. The exponent determines the number of times the base is multiplied by itself. For example: $3^2 = (3)(3) = 9$.

Rational exponents are exponents that are in fraction form. The following are examples of expressions with rational exponents.

$$81^{\frac{1}{4}}; \quad 25^{\frac{3}{5}}; \quad 9^{\frac{2}{3}}$$

Expressions with rational exponents can be written in radical form. The numerator is the number of times the base, which is placed under the radical sign, is multiplied by itself. The denominator is the root of the radical expression. A root is the inverse of an exponent. The examples below show how expressions with rational exponents are written in radical form.



More Examples:

$$9^{\frac{2}{3}} = \sqrt[3]{9^2}$$

$$25^{\frac{3}{5}} = \sqrt[5]{25^3}$$

A root of 2 is called a square root and is the

inverse of the exponent 2. A root of 3 is called a cube root and is the inverse of the exponent 3. The following examples show how to simplify expressions with rational exponents in two ways.

Simplifying by converting to radical form:

Example 1: Simplify.

$$8^{\frac{1}{3}}$$

(1) $8^{\frac{1}{3}} = \sqrt[3]{8^1}$

(2) $\sqrt[3]{8}$

(3) $\sqrt[3]{(2)(2)(2)}$

(4) $\sqrt[3]{\underbrace{(2)(2)(2)}_{2}}$

Step 1: Rewrite the rational expression in radical form.
Step 2: Simplify: 8 to the first power equals 8.
Step 3: Factor 8 into (2)(2)(2).
Step 4: Since there are three factors of 2 under the cube root, the radical expression will simplify to 2.

Answer: 2

Simplifying using exponent laws:

(1) $8^{\frac{1}{3}} = [(2)(2)(2)]^{\frac{1}{3}}$

(2) $[2^3]^{\frac{1}{3}}$

(3) $2^{3\times\frac{1}{3}}$

2^1

2

Step 1: Rewrite the expression and factor 8 into (2)(2)(2).
Step 2: Write (2)(2)(2) in exponential form.
Step 3: Multiply 3 and 1/3 (remember that when an exponent is taken to another power, the two powers are multiplied) to get 1. Simplify 2 to the first power to 2.

Answer: 2

Example 2: Simplify.

$$32^{\frac{2}{5}}$$

(1) $32^{\frac{2}{5}} = \sqrt[5]{32^2}$

(2) $\sqrt[5]{1,024}$

(3) $\sqrt[5]{(4)(4)(4)(4)(4)}$

(4) $\sqrt[5]{\underbrace{(4)(4)(4)(4)(4)}_{4}}$

Step 1: Rewrite the expression as a radical expression.
Step 2: Simplify 32^2 : 32 × 32 = 1,024.
Step 3: Factor 1,024 into (4)(4)(4)(4)(4). Always try to factor the number under the radical into the same number of factors as the root.
Step 4: Since this is a 5th root and there are five 4s under the radical, the radical expression will simplify to 4.

Answer: 4

Activities that can help reinforce the concept of rational exponents are as follows.

1. Take 30 index cards, and make three piles of ten cards each. Write expressions with rational exponents on the first set of ten cards. On the second set, write the radical form for each expression you wrote on the first ten cards. On the third set, write the simplified answer to the ten expressions. Shuffle all of the cards and lay them face down on a table. Each player takes turns flipping three cards. If all three cards go together, the player has made a match and gets to try again. If not, it is the next player's turn.

2. Name expressions with rational roots, and ask the student to factor the base into the same number of identical factors as the root. For example, if the root is 1/6, the student should factor the base into 6 identical factors, so that when the factors are rewritten in exponential form, and the

exponent is multiplied by the root, the product is 1.

Polynomials: Subtraction

A monomial is the product of a number and an unknown variable or unknown variables. 6xy is a monomial. The sum or difference of two or more monomials is called a polynomial.

Here is an example of a polynomial:

$y^2 + 4y + 3$.

Adding and subtracting polynomials includes simplifying and combining "like" terms. Like terms are monomials that have the same variable or variables for which the variable or variables have the same exponent.

Examples :

$\left\{\begin{matrix} 2x \\ 4x \end{matrix}\right\}$ like terms $\quad \left\{\begin{matrix} 2x \\ -4x^2 \end{matrix}\right\}$ not like terms

To subtract polynomials, first write the polynomials as one long polynomial. Then distribute the subtraction sign through the second polynomial. Finally, combine like terms. Practice by subtracting the following polynomials.

Example 1:

Subtract $(p^2 - 2p - 6)$ from $(p^2 + 3p + 3)$.

(1)

$p^2 + 3p + 3 - (p^2 - 2p - 6)$

(2)

(p^2) becomes

$(-2p)$ becomes

(-6) becomes

(3)

$p^2 + 3p + 3 - p^2 + 2p + 6$

(4)

$p^2 - p^2 = 0$

$3p + 2p = 5p$

$3 + 6 = 9$

Step 1: Set up the two polynomials as one long polynomial. Since the problem is to subtract one polynomial from another, the second polynomial in the problem must be written first.

Step 2: Distribute the subtraction sign through the second polynomial. This involves changing the sign of each term in the second polynomial.

Step 3: Rewrite the polynomial after changing the signs in the second polynomial.

Step 4: Combine like terms.

Answer: 5p + 9

Example 2: Subtract four times a number decreased by ten from eight times the same number less six.

Step 1: "Four times a number decreased by ten" can be written (4x - 10).

Step 2: "Eight times the same number less six" can be written (8x - 6).

Step 3: Now the problem reads: Subtract (4x - 10) from (8x - 6).

(4)
$(8x - 6) - (4x - 10)$

(5)
$4x$ becomes $-4x$
-10 becomes $+10$

(6)
$8x - 6 - 4x + 10$

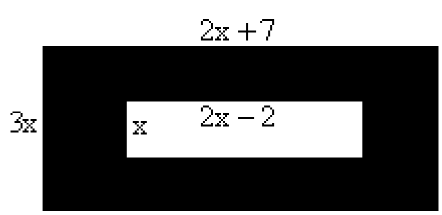
(7)
$8x - 4x = 4x$
$-6 + 10 = 4$

Step 4: Set up the polynomials as one long polynomial.
Step 5: Distribute the subtraction sign through the second polynomial. This involves changing the sign of each term in the second polynomial.
Step 6: Rewrite the entire polynomial after changing the signs in the second polynomial.
Step 7: Combine like terms.

(4x - 10) subtracted from (8x - 6) equals 4x + 4.

Answer: 4x + 4

Example 3: Find area of the shaded region.



(1)
$3x(2x + 7)$
$3x(2x) + 3x(7)$
$6x^2 + 21x$

(2)
$x(2x - 2)$
$x(2x) - x(2$
$2x^2 - 2x$

(3)
$(6x^2 + 21x) - (2x^2 - 2x)$

(4)
$6x^2 + 21x - 2x$

(5)
$6x^2 - 2x^2 = 4x^2$
$21x + 2x = 23x$

Step 1: Determine the area of the large rectangle by multiplying the length (2x + 7) by the width (3x). This involves multiplying each term in (2x + 7) by 3x.
Step 2: Determine the area of the small rectangle by multiplying the length (2x - 2) by the width (x). This involves multiplying each term in (2x - 2) by x.
Step 3: Now subtract the area of the small rectangle from the area of the large rectangle. Remember to put the second polynomial in parentheses since this is subtraction.
Step 4: Distribute the subtraction sign through the second polynomial. This involves changing the sign of each term in the second polynomial.
Step 5: Combine like terms.

Answer: $4x^2 + 23x$.

Equations: Order of Operations
An equation is a statement in which two numbers or two expressions are set equal to each other. For

example,
5 + 3 = 8 and 16 = 3c + 4 are equations.

When solving equations, find the value of the variable by getting the variable alone on one side of the equal sign. To do this, undo any operations on the variable by using the inverse operation. Any operation done on one side of the equal sign must be done on the other side of the equal sign in order to keep the statement true.
If a number has been added to the variable, subtract the number from both sides of the equation.

$$\begin{array}{rcr} m + 3 & = & 5 \\ -3 & & -3 \\ \hline m & = & 2 \end{array}$$

If a number has been subtracted from the variable, add the number to both sides of the equation.

$$\begin{array}{rcr} b - 7 & = & 9 \\ +7 & & +7 \\ \hline b & = & 16 \end{array}$$

If a variable has been multiplied by a nonzero number, divide both sides by the number.

$$6c = 12$$
$$\frac{6c}{6} = \frac{12}{6}$$
$$c = 2$$

If a variable has been divided by a number, multiply both sides by the number.

$$\frac{c}{2} = 6$$
$$\frac{c}{2} \times 2 = 6 \times 2$$
$$c = 12$$

When solving 2-step equations, we must first undo the addition or subtraction using the inverse operation, then undo the multiplication or division:

$$2n - 6 = 8$$
$$\begin{array}{rcr} 2n - 6 & = & 8 \\ +6 & & +6 \\ \hline \end{array}$$
$$\frac{2n}{2} = \frac{14}{2}$$
$$n = 7$$

Example 1: Solve the equation for t.
5(t - 4) = t + 12

$$5(t - 4) = t + 12$$
$$5t - 20 = t + 12$$

Step 1: Multiply 5 times the terms inside the parenthesis.

$$\begin{array}{rcr} 5t - 20 & = & t + 12 \\ +20 & & +20 \\ \hline 5t & = & t + 32 \end{array}$$

Step 2: Add 20 to both sides of the equation

$$\begin{array}{rcr} 5t & = & t + 32 \\ -t & & -t \\ \hline 4t & = & 32 \end{array}$$

Step 3: Subtract t from both sides of the equation

$$\frac{4t}{4} = \frac{32}{4}$$
$$t = 8$$

Step 4: Divide both sides of the equation by 4.

Answer: t = 8

Example 2: Evaluate the expression for c = 3:
2(c + 4) + 2(15)

(1) 2(3 + 4) + 2(15)
(2) 2(7) + 2(15)
(3) 14 + 30
(4) 44

Step 1: Substitute 3 in place of 'c' in the expression.
Step 2: Add the numbers in parentheses.
Step 3: Rewrite the equation after performing all multiplications in order from left to right.

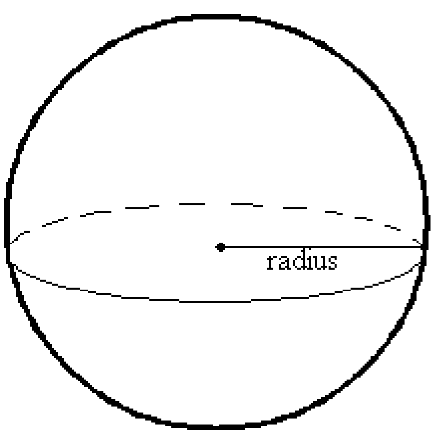
Step 4: Add 14 and 30 to get 44.

Answer: 44

Volume of Spheres
This study guide will focus on finding the volume of a sphere.

Remember the volume is the measure of the amount of space inside an object.

A sphere is a three-dimensional circle.



Sphere

When a student is asked to find the volume of a sphere, he or she will be given the radius, the distance between the center of a sphere and any point on the sphere , or the diameter, the distance between any two points on the sphere passing through the center. They will also need the following formula which may or may not be given:

$$V = \frac{4}{3}\pi r^3 \quad \text{where } \pi = 3.14$$

$$r = \text{radius}$$

To solve, substitute the radius (divide the diameter by two to get the radius) for *r* and 3.14 for π. Then use order of operations to find the answer. A quick review of order of operations is provided below.

P E M D A S

P = Parentheses, E = Exponents, M = Multiplication, D = Division, A = Addition, S = Subtraction.
**Note: Evaluate multiplication and division from left to right (whichever comes first). Evaluate addition and subtraction from left to right (whichever comes first).

Example 1: Calculate the volume of a sphere with radius 9.5 m. Round to the nearest cubic meter. Use π = 3.14.

(1)

$$V = \frac{4}{3}(3.14)(9.5)^3$$

(2)

$$V = \frac{4}{3}(3.14)(857.375)$$

(4)

$$V = 3{,}590 \text{ m}^3$$

Step 1: Substitute 9.5 m in for the radius

and 3.14 for π.
Step 2: Evaluate the exponent first by taking 9.5 to the third power to get 857.375.
Step 3: Multiply across to get 3,589.543...
Step 4: Round to the nearest cubic meter to get 3,590 m^3.

Answer: 3,590 m^3

Example 2: Find the volume of a baseball that is 2.8 inches in diameter. Round your answer to the nearest cubic inch.

(1) $\frac{2.8}{2} = 1.4$

(2) $V = \frac{4}{3}(3.14)(1.4)^3$

(3) $V = \frac{4}{3}(3.14)(2.744)$

(4) $V = 11.48821\overline{3}\ \text{in}^3$

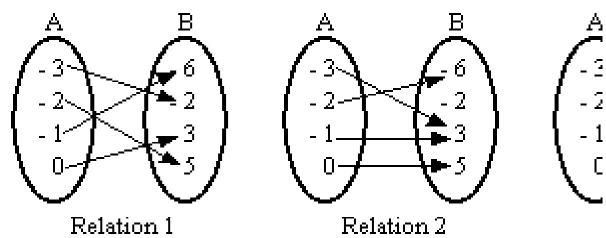
(5) $V = 11\ \text{in}^3$

Step 1: Find the radius. Remember that the radius is half of the diameter so 2.8 ÷ 2 = 1.4 inches.
Step 2: Substitute 1.4 for r and 3.14 for π.
Step 3: Evaluate the exponent first by taking 1.4 to the third power to get 2.744.
Step 4: Multiply across to get 11.4882333...
Step 5: Round to the nearest cubic inch to get 11 in^3.

Answer: 11 in^3

An activity that would help reinforce this skill would be to have the student research the different diameters or radii of spherical objects like sporting equipment or the planets. Have him or her estimate the volume of the different objects. Then have the student verify his or her prediction by calculating the actual volume using the formula.

Functions/Relations - A
A relation is a set of ordered pairs that represent a relationship between the elements of the two sets. A function is a special type of relation, where each element of the first set (x-values) corresponds to an unique element of the second set (y-values). The first set of numbers is commonly known as the input and the second set as the output. The input, or x-values, are entered into the equation. Once evaluated, the result is the output, or y-values. In other words, in order for a relation to be a function, for each x-value there can be no more than one value of y. Some examples of relations are given below, with input values in A mapped to output values in B.



Relations 1 and 2 are functions, while relation 3 is not a function. The input value - 1 in relation 3 is matched to more than one output value (3 and 5), so the relation is not a function.

Example 1:
Which of the following relations is not a function?

(A) {(6, - 9), (12, 4), (- 10, - 3), (4, 12)}

(B) {(7, - 10), (4, 4), (- 7, 10), (11, - 5)}

(C) {(9, - 1), (- 12, - 1), (9, 4), (15, -11)}

(D) {(7, - 1), (9, -14), (13, - 5), (- 5, - 1)}

Solution:
If there is a value of x resulting in more than one value of y, the relation is not a function. This only occurs in the third set of numbers with (9, - 1) and (9, 4). Therefore, set C is not a function.

Answer: Set C is not a function.

Example 2:
Which of the following points, if removed from the set, would make the set a function?

{(- 4, 5), (4, - 5), (- 4, 4), (- 5, 5), (5, - 4)}

Solution:
The ordered pairs (- 4, 5) and (- 4, 4) have the same x values but different y values. Therefore, if either point is removed from the set, the remaining ordered pairs will represent a function.

Answer: Remove either (- 4, 5) or (- 4, 4).

Mutually Exclusive/Inclusive
Two events are said to be mutually exclusive if they cannot occur at the same time. Examples of mutually exclusive events are: rolling a 3 and a 4 at the same time when rolling one die OR tossing one coin and having it land on heads and tails at the same time. These examples show situations that cannot happen at the same time because if one happens, the other has no chance of occurring. A special formula is needed to determine the probability of mutually exclusive events.

P(A *or* B) = P(A) + P(B), where A and B are mutually exclusive events

If two events are not mutually exclusive, they are said to be inclusive and could possibly occur at the same time. For these cases, a more general formula must be used.

P(A *or* B) = P(A) + P(B) - P(A *and* B), where A and B are any events

It is important to note that if it cannot be determined that A and B are mutually exclusive, use the formula for inclusive events because it will work even if A and B are mutually exclusive.

Example 1: Javier is on a television game show. In order to win the $25,000.00 showcase, he must draw a king or a queen from a standard deck of cards. What is the probability that Javier will win the showcase?

(1) $P(\text{king}) = \frac{4}{52} = \frac{1}{13}$

(2) $P(\text{queen}) = \frac{4}{52} = \frac{1}{13}$

(3) $P(\text{king or queen}) = P(\text{king}) + P(\text{queen}) = \frac{1}{1}$

Step 1: These events are mutually exclusive because it is not possible to draw a king and a queen out of a standard deck of cards if you only draw one card. Determine the probability of drawing a king from the deck of cards. There are 4 kings in a deck of cards and a total of 52 cards in the deck, so the probability of drawing a king is 4/52 or 1/13.
Step 2: Determine the probability of drawing a queen from the deck of cards. There are 4 queens in a deck of cards and a total of 52 cards in the deck, so the probability of drawing a queen is 4/52 or 1/13.
Step 3: Use the formula for mutually exclusive events. Fill in the correct probabilities and add.

Answer: $P(\text{king or queen}) = \frac{2}{13}$

Example 2: Michele is competing in a swim meet. The probability that she will win her first race is 0.82 and the probability that she will win her second race is 0.23. If the probability that she will win both of her races is 0.19, what is the probability that Michele will win either the first or second race?

(1) P(first) = 0.82
P(second) = 0.23
P(first *and* second) = 0.19

(2) P(first *or* second) = P(first) + P(second) - P(first *and* second)
P(first *or* second) = 0.82 + 0.23 - 0.19 = 0.86

Step 1: Use the formula for inclusive events, since it is possible that she could win both races. Determine the probabilities needed for the formula. P(first), P(second), and P(first *and* second).
Step 2: Substitute the correct probabilities into the formula for inclusive events and simplify.

Answer: 0.86

As an extension activity, have the student make up probabilities for events from his or her life such as tests, chores, games, etc. Use these probabilities to make up scenarios using the formulas for mutually exclusive and inclusive events. (To determine P(A *and* B), multiply the probability of A by the probability of B. In other words, P(A *and* B) = P(A) × P(B).)

Theoretical Probability - B

Probability is the likelihood that a given event will occur. There are two types of probabilities: theoretical and experimental. Theoretical probability is a strictly mathematical probability. It is determined by taking the ratio of the number of ways an event can occur to the total number of possibilities in the sample space. Experimental probability is a probability that is determined using the results of an experiment. It is determined by taking the ratio of the number of times the event occurred in the experiment to the total number of possibilities in the sample space. This study guide will focus on theoretical probability.

To determine the theoretical probability of an event:

The probability or "chance" of an event is determined by two groups of outcomes. The first is how many outcomes are possible in a probability situation, such as the rolling of a die or tossing a coin. In the case of rolling a six-sided fair die, there are six possible outcomes, $\{1, 2, 3, 4, 5, 6\}$.
The group or set of all the possible outcomes is called the sample space for an experiment. The second group of outcomes to be determined is a subgroup or subset of the sample space called the event. For example, if a six-sided fair die is rolled, the event that the outcome is an odd number would have three possible outcomes $\{1, 3, 5\}$.
The probability of an event, *E*, denoted P(*E*), is determined by the following formula:

$$P(E) = \frac{\text{number of elements in the event}}{\text{number of elements in the sample spa}}$$

where S represents sample space.

Note that the above formula is only valid in situations where each outcome in the sample space is equally likely to occur, such as when a six-sided fair die is rolled. Any of the six possible outcomes are just as likely to occur as another. In addition, if the number of elements in the event is the same as the number of elements in the sample space, that is n(E) = n(S), then P(E) = 1.

Example 1: If a six-sided fair die is rolled, find the probability of rolling an odd number.

(1) S = {1,2,3,4,5,6}
Therefore, n(S) = 6

(2) E = {1,3,5}
Therefore, n(E) = 3

$$\textbf{(3) } P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Step 1: There are six possible outcomes when a six-sided fair die is rolled. Thus, the number of elements in the sample space is 6, so n(S) = 6.
Step 2: There are three possible outcomes of the event of rolling an odd number. The number of elements in the event is 3, so n(E) = 3.
Step 3: Substitute the values of n(E) and n(S) in the probability formula, 3 and 6, respectively. Then simplify the fraction.

Answer: 1/2 or 0.5

The sum of the probabilities of a sample space:
The sum of all the probabilities of a sample space is one. For example, if a six-sided fair die is rolled, the probability of rolling any number 1 - 6 is 1/6 (or 0.16 where the 6 repeats). The sum of the probabilities is found below.

$$P(1) = \frac{1}{6}, \quad P(2) = \frac{1}{6}, \quad P(3) = \frac{1}{6},$$
$$P(4) = \frac{1}{6}, \quad P(5) = \frac{1}{6}, \quad P(6) = \frac{1}{6},$$

$$P(1,2,3,4,5,6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

Using this information, it can be said that if the probability of an event occurring is P(E), then the probability of the same event NOT occurring is 1 - P(E).

Example 2: The probability of seeing a blue jay in December is 0.05. What is the probability of NOT seeing a blue jay in December?

Solution: Use the sum of probabilities formula to determine the necessary probability. P(E) = 0.05, P(not E) = 1 - P(E). Substitute the value of P(E) into the formula: 1 - 0.05 = 0.95.

Answer: The probability of NOT seeing a blue jay in December is 0.95.

Applying theoretical probability in real world situations:
Once the student is comfortable with the idea of theoretical probability and the sum of the probabilities of an event, he or she will be ready to apply probability to real world situations.

Example 3: The probability of Babe Ruth

hitting a home run when up to bat was 0.085. If he got up to bat 5,000 times, how many home runs would he be expected to hit? Round your answer to the nearest whole number, if necessary.

$$(1)\ \ 0.085 \times 5{,}000 = 425$$

<u>Step 1</u>: To determine the number of expected home runs, multiply the probability of hitting a home run (0.085) by the number of times he was up to bat (5,000).

Answer: Babe Ruth would be expected to hit 425 home runs.

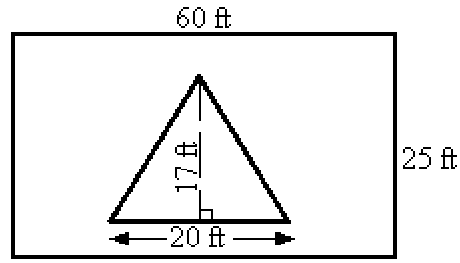
Example 4: A light bulb manufacturing company has determined that the probability that a brand new light bulb will NOT work is 0.013. If the company shipped 7,235 light bulbs yesterday, how many of the light bulbs can be expected to work? Round your answer to the nearest whole number.

$$(1)\ \ 1 - 0.013 = 0.987$$
$$(2)\ \ 0.987 \times 7{,}235 = 7{,}140.945 \approx 7{,}141$$

<u>Step 1</u>: Determine the probability that a light bulb WILL work by subtracting the probability that it will NOT work from 1.
<u>Step 2</u>: Multiply the probability that a light bulb will work by the number of light bulbs shipped and round the answer to the nearest whole number.

Answer: 7,141 light bulbs would be expected to work.

Example 5: Evan and his teammates are participating in the egg drop at their school. The egg will be dropped from the top of the school building and it needs to land in a triangular region of the grassy area in the schoolyard. The schoolyard is a rectangular area that is 60 feet long and 25 feet wide. The landing zone is a triangle with a height of 17 feet and a base of 20 feet. What is the probability that the egg will land in the triangular landing zone? Round your answer to the nearest hundredth, if necessary.



$$(1)\ \ 60\text{ ft} \times 25\text{ ft} = 1{,}500\text{ ft}^2$$
$$(2)\ \ \frac{1}{2}(20\text{ ft})(17\text{ ft}) = 170\text{ ft}^2$$
$$(3)\ \ \frac{170\text{ ft}^2}{1{,}500\text{ ft}^2} = 0.11\overline{3} \approx 0.11$$

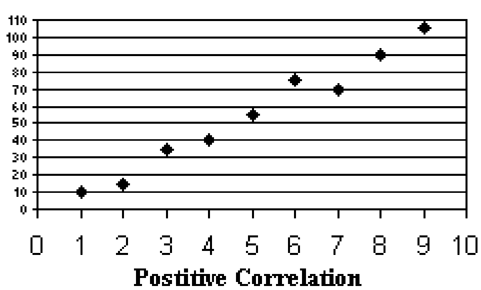
<u>Step 1</u>: Determine the area of the rectangular schoolyard (the entire area where the egg could land).
<u>Step 2</u>: Determine the area of the triangular landing zone (the area where they want the egg to land).
<u>Step 3</u>: The probability of landing in the landing zone can be determined by finding the probability that the egg will land in the correct amount of area. Divide the area of the triangular landing zone by the area of the rectangular schoolyard and round the answer to the nearest hundredth.

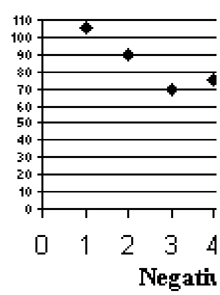
Answer: The probability that the egg will land in the triangular landing zone is 0.11.

An activity that could help to reinforce this skill is to look at the sports section of a newspaper to find player statistics. Then have the student make predictions using this data. For example, have the student predict the number of times a basketball player will or will not score baskets based on a given number of tries.

Estimate Line of Best Fit: Scatter Plot

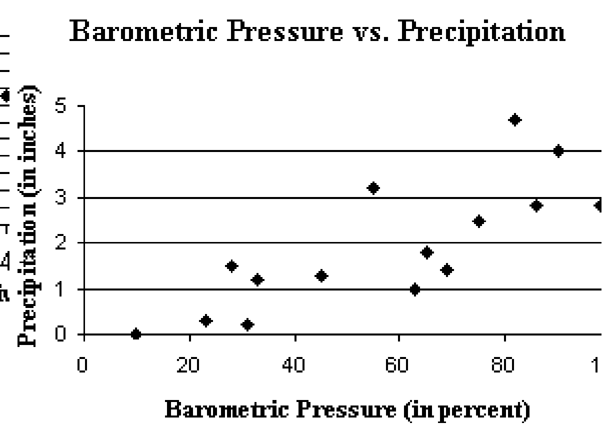
A scatter plot is a graph that shows how much one variable is affected by another. One variable is plotted along the horizontal axis and the other variable is plotted along the vertical axis. The relationship between the two variables is called their correlation. Scatter plots usually consist of a large amount of data. The closer the data points come to making a straight line, the higher the correlation between the two variables (or the stronger the relationship). If the data points fall along a straight line from a low value to a high value, then the values are said to have a positive correlation. If the data points fall along a straight line going from a high value to a low value, then the variables are said to have a negative correlation. When looking for positive or negative correlation, look at the graph from left to right.





A line of best fit shows the relationship between two variables in a scatter plot. The line that is drawn represents an *average* of all of the data points, most of which will probably not lie on the line itself. Remember, the line is an estimation and not an exact measure. The points on each side of the line of best fit should be as close to the line as possible. Once the line of best fit is drawn, it is possible to estimate the value of one variable if the value of the other variable is known. This study guide will focus on drawing an estimated line of best fit for a scatter plot.

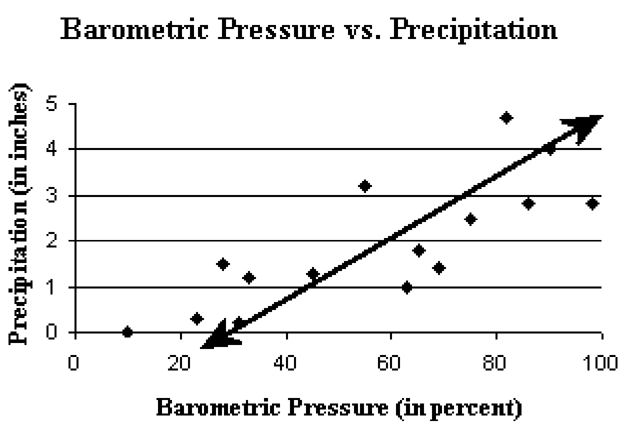
Example 1: Draw an estimated line of best fit for the Barometric Pressure vs. Precipitation scatter plot below.



Step 1: Determine if there is a positive or a negative correlation between the two variables. Since the precipitation increases as the barometric pressure increases, there is

a positive correlation. This means that the line of best fit will go up from left to right.

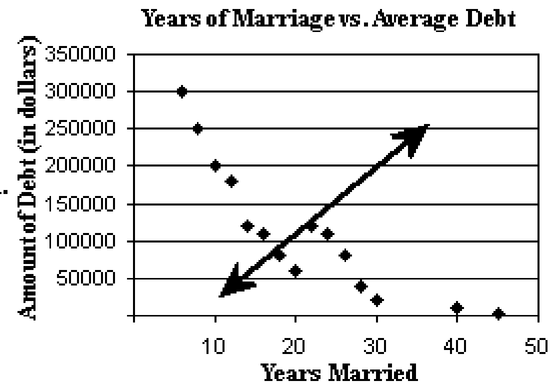
Step 2: Determine where to place the line of best fit. Recall that the line of best fit represents the *average* of all of the data points. This means that there should be close to the same number of data points above the line of best fit as there are below the line of best fit, and the points on each side of the line should be as close to the line as possible.



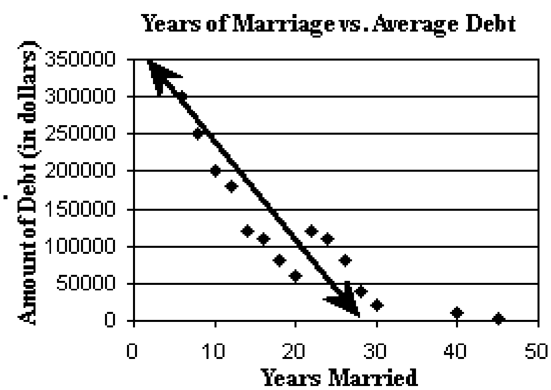
The placement of this line of best fit includes 7 points above the line and 6 points below the line. This is the estimated line of best fit.

Example 2: The graphs below show the relationship between the number of years a couple has been married and the amount of debt the couple is in. Which graph illustrates the line of best fit for this data?

A.



B.



Step 1: Determine whether there is a positive or a negative correlation between the two variables. Since the data points in this case get lower as the graph is read from left to right, there is a negative correlation. This means that the line of best fit will go down from left to right.

Step 2: Eliminate answer choices **A.** and **C.** because their lines of best fit do not go down from left to right.

Step 3: Of the two graphs left, determine which one represents the *average* of the two variables. Remember that this means that there should be approximately the same number of data points above the line of best fit as there are below the line of best fit and the points on each side of the line should be as close to the line as possible.

Answer: B. This graph shows the data points with a line of best fit that is drawn in

the correct direction and has approximately the same number of data points above the line and below the line.

An activity that could help the student with this skill is to find scatter plots in newspapers, magazines, and on the Internet and have the student draw estimated lines of best fit for the data. Have the student justify the reasoning behind drawing the line by asking questions such as:
• Why did he or she draw a positive (or negative) correlation?
• Why did he or she place the line where it is?
• What does the line of best fit tell about the data?

Compound Interest

Compound interest is the interest paid on the initial deposit as well as any interest previously earned. Interest is usually compounded (paid or figured) more than one time per year.

Biannually (Semiannually) = compounded twice a year

Quarterly = compounded four times a year

Monthly = compounded twelve times a year

Daily = compounded 365 times a year

There is a special formula for determining compound interest.

Compound Interest Formula

$$S = P\left(1 + \frac{r}{n}\right)^{nt}$$

The initial deposit is called the principal and is denoted with the variable *P*. The interest rate, denoted with the variable *r*, is the percent of the principal that will be earned over a particular period of time, usually one year. The number of times interest is compounded per year is denoted by the variable *n*. The variable *t* states the number of years (time) the principal is invested. The variable *S* in the formula represents the amount of money the account will hold after the interest is accrued over the stated number of years.

Example 1: If $5,000.00 is invested in an account paying an annual interest rate of 8%, find the amount of money in the account after 5 years if the interest is compounded quarterly.

(1) $P = \$5000 ;\ r = 8\% = 0.08 ;\ n = 4 ;\ t = 5$

(2) $S = 5000\left(1 + \frac{0.08}{4}\right)^{(4)(5)}$

(3) $S = 5000(1.02)^{20}$

(4) $S = \$7429.74$

Step 1: The principal (initial deposit) is $5,000, so *P* = $5000. The interest rate is 8%. The interest rate must be converted from a percent into a decimal, so *r* = 0.08. The interest is compounded quarterly (4 times per year), so *n* = 4. The money will be in the account for 5 years, so *t* = 5.
Step 2: Substitute the known values in place of the variables that represent them.
Step 3: Divide 0.08 by 4, then add that result to 1. Next, 4 and 5 must be multiplied to come up with the exponent of 20.
Step 4: Applying the order of operations, take 1.02 to the 20th power. This can be accomplished using a calculator. First, enter 1.02. Then, press the x^y key, y^x key, or the ^ key. The next step is to enter the number 20, and the final step is to press the equal sign. 1.02 to the 20th power equals 1.485947396. Do not round this number! To finish this problem, multiply 1.485947396 by 5000.

Answer: The amount of money in the account after 5 years is $7,429.74.

Example 2: Genevieve bought an airline ticket for $315.00 with a credit card that charges an annual rate of 17.9% and compounds the interest monthly. If she does not pay for the ticket for seven months, how much will she have paid for the ticket?

(1) $P = \$315$; $r = 17.9\% = 0.179$; $n = 12$; and $t = \frac{7}{12}$
(2) $S = 315\left(1 + \frac{0.179}{12}\right)^{12 \cdot \frac{7}{12}}$
(3) $315(1.014916666)^7$
(4) $349.40

Step 1: The principal (initial amount invested) is $315.00, so P = $315. The interest rate is 17.9%, so $r = 0.179$. The interest is compounded monthly and there are 12 months in one year, so $n = 12$. The airline ticket is not going to be paid off for 7 months, so the amount of time is 7/12 of a year ($t = 7/12$).
Step 2: Substitute the known values in place of the variables that represent them.
Step 3: Divide 0.179 by 12, then add the result to the 1. (Note: Do not round until all calculations are complete.) Next, the 12 and the 7/12 must be multiplied to come up with the exponent of 7.

$$\frac{12}{1} \times \frac{7}{12} = \frac{84}{12} = \frac{7}{1} = 7$$

Step 4: Calculate 1.014916666 to the 7th power, which equals 1.109207228. (Do not round yet.) To finish this problem, multiply 1.109207228 by 315.

Answer: $349.40

Example 3: How much principal must the Jacob family invest now at 3% compounded monthly in order to have enough money to buy a $25,000.00 SUV in 5 years?

(1) $S = \$25{,}000;\ r = 3\%;\ n = 12;\ t = 5$
(2) $25{,}000 = P\left(1 + \frac{0.03}{12}\right)^{(12)(5)}$
(3) $25{,}000 = P(1.0025)^{60}$
(4) $25{,}000 = P(1.161616782)$
(5) $\frac{25{,}000}{1.161616782} = \frac{P(1.161616782)}{1.161616782}$
(6) $P = \$21{,}521.73$

Step 1: This problem asks the student to solve for P instead of S, so the student should pay close attention when substituting the values into the formula. The ending amount after the interest is accrued is $25,000.00, so $S = 25{,}000$. The interest rate is 3%, so $r = 0.03$. The interest is compounded monthly and there are 12 months in one year, so $n = 12$. The money will be invested for 5 years, so $t = 5$.
Step 2: Substitute the known values into the formula.
Step 3: Divide 0.03 by 12 and add 1 to the result. Next, 12 and 5 must be multiplied to come up with the exponent of 60.
Step 4: Raise 1.0025 to the 60th power using a calculator. The result is 1.161616782.
Step 5: Since this problem needs to be solved for P, divide both sides of the

equation by 1.161616782 to isolate the variable, P.
Step 6: The result is 21,521.72764 which is approximately equal to \$21,521.73.

Answer: \$21,521.73

Continuously Compounded Interest:

Interest can also be compounded continuously. There is a special formula to follow when compounding interest continuously.

Continuously Compounded Interest Formula

$$A = Pe^{rt}$$

The variable A in the formula is the amount in the account after a certain number of years. The variable P is the principal, r is the annual interest rate, and t is the number of years. The variable e represents the constant irrational number 2.718281828459... There is a special key on scientific calculators for calculating e taken to a power. That key is: e^x

Example 4: Calculate $e^{0.05}$.

(1) Enter 0.05.
(2) Press e^x

OR

(1) Press e^x (2) Enter 0.05
(3) Press = or ENTER

Answer: approximately 1.051271

Example 5: If Michael invests \$900.00 at an annual interest rate of 6% compounded continuously, what is the amount in the account after 10 years?

(1) $P = 900$; $r = 0.06$; and $t = 10$
(2) $A = 900\,e^{(0.06)(10)}$
(3) $A = 900\,e^{0.6}$
(4) $A = \$1{,}639.91$

Step 1: The principal, P, is 900 because that was Michael's initial deposit. The interest rate is 6%, so $r = 0.06$, and the money will be in the account for 10 years, so $t = 10$.
Step 2: Substitute the known values in place of the variables that represent them.
Step 3: Multiply 0.06 by 10 to determine the exponent for e.
Step 4: Calculate $e^{(0.6)} = (1.8221188)$. Multiply 1.822
Remember, do not round until all calculations are complete.
Answer: \$1,639.91

An activity to help reinforce this concept is to have the student calculate how much money he or she would have in a savings account using various interest rates, principals, and times. To make the activity more realistic, students could use the Internet to research available interest rates at various local banks.

Theoretical Probability - A

Theoretical probability is strictly mathematical probability. There is no experimentation involved. The theoretical probability of an event is figured using a calculated formula.

To determine the probability of an event:

The probability or "chance" of the occurrence of an event is determined by two groups of outcomes. The first is how many outcomes are possible in an experiment, such as the rolling of a die or tossing a coin. In the case of rolling a six-sided fair die, there are six possible outcomes, $\{1, 2, 3, 4, 5, 6\}$.
The group or set of all the possible outcomes is called the sample space for an experiment. The second group of outcomes to be determined is a subgroup or subset of the sample space called the event. For example,

if a six-sided fair die is rolled, the event that the outcome is an odd number would have 3 possible outcomes $\{1, 3, 5\}$.
The probability of an event, E, denoted P(E), is determined by the following formula:

$$P(E) = \frac{\text{number of elements in the event}}{\text{number of elements in the sample space}} = \frac{n(E)}{n(S)}$$

where S represents sample space.

Note that the above formula is only valid in situations where each outcome in the sample space is equally likely to occur, such as when a six-sided fair die is rolled. Any of the six possible outcomes are just as likely to occur as another. In addition, note that if the number of elements in the event is the same as the number of elements in the sample space, that is n(E) = n(S), then P(E) = 1.

Example 1: If you are rolling a six-sided fair die, find the probability of rolling an odd number.

(1) $S = \{1,2,3,4,5,6\}$
Therefore, $n(S) = 6$

(2) $E = \{1,3,5\}$
Therefore, $n(E) = 3$

(3) $P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

Step 1: There are six possible outcomes when a six-sided fair die is rolled. Thus, the number of elements in the sample space is 6, so n(S) = 6.
Step 2: There are three possible outcomes of the event of rolling an odd number. The number of elements in the event is 3, so n(E) = 3.
Step 3: Substitute the values of n(E) and n(S) in the probability formula, by 3 and 6, respectively. Then simplify the fraction.

The probability of rolling an odd number is 1/2.

Example 2: If you are rolling a six-sided fair die, find the probability of NOT rolling a four.

(1) $S = \{1,2,3,4,5,6\}$
Therefore, $n(S) = 6$

(2) $E = \{1,2,3,5,6\}$
Therefore, $n(E) = 5$

(3) $P(E) = \frac{n(E)}{n(S)} = \frac{5}{6}$

Step 1: There are six possible outcomes when a six -sided fair die is rolled. Thus the number of elements in the sample space is 6, so n(S) = 6.
Step 2: There are five possible outcomes of the event of not rolling a four. The number of elements in the event is 5, so n(E) = 5.
Step 3: Substitute the values of n(E) and n(S) in the probability formula, by 5 and 6, respectively.

The probability of not rolling a four is 5/6.

Determining the sample space when rolling two six-sided fair die:

When rolling two six-sided fair die, there are thirty-six possible outcomes. One possible outcome is to get a 1 on the first roll, followed by a 4 on the second roll. To denote this outcome, it is helpful to use the notation (1, 4). All thirty-six outcomes are listed in the chart below.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Example 3: If you are rolling two six-sided fair die, find the probability of rolling numbers totaling the sum of 8.

$$(1)\ n(S) = 36$$

$$(2)\ E = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$
$$\text{Therefore, } n(E) = 5$$

$$(3)\ P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$$

Step 1: There are thirty-six possible outcomes as indicated by the chart. Thus the number of elements in the sample space is 36, so n(S) = 36.
Step 2: There are five possible outcomes of the event of rolling numbers totaling the sum of 8. In each of the listed ordered pairs, the sum of the two components is 8 such as 2 + 6 = 8. Thus the number of elements in the event is 5, so n(E) = 5.
Step 3: Substitute the values of n(E) and n(S) in the probability formula, by 5 and 36, respectively.

The probability of rolling numbers totaling 8 is 5/36.

Example 4: You have a container filled with five pieces of green paper and seven pieces of white paper. What is the probability of choosing a white piece of paper?

$$(1)\ n(S) = 12$$

$$(2)\ n(E) = 7$$

$$(3)\ P(E) = \frac{n(E)}{n(S)} = \frac{7}{12}$$

Step 1: There are 12 possible outcomes in this problem because there are 12 total pieces of paper that can be picked since 5 + 7 = 12. Thus the number of elements in the sample space is 12, so n(S) = 12.
Step 2: There are seven possible outcomes of choosing a white piece of paper because there are seven white pieces of paper. Thus the number of elements in the event is 7, so n(E) = 7.
Step 3: Substitute the values of n(E) and n(S) in the probability formula, by 7 and 12, respectively.

The probability of choosing a white paper is 7/12.

Note that the probability of choosing a green piece of paper would be 5/12 because the number of elements in the event would be 5 since there are five pieces of green paper.

Determining the sample space for a standard deck of cards:

A standard deck of cards contains 52 cards, thus n(S) = 52 if the event requires picking cards out of a standard deck. A deck is composed of four different suits: spades, clubs, diamonds, and hearts. Each suit contains 13 cards (2 - 10, a jack, a queen, a king, and an ace). The suits of diamonds and hearts are red cards and the suits of spades and clubs are black cards.

Example 5: If you were to draw a card from a standard deck, what is the probability of drawing a five?

$$(1)\ n(S) = 52$$

$$(2)\ n(E) = 4$$

$$(3)\ P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

Step 1: There are 52 cards in a standard deck, thus the number of elements in the sample space is 52, so n(S) = 52.
Step 2: There are four possible ways to draw a five: the five of spades, the five of clubs, the five of diamonds, and the five of hearts. Thus the number of elements in the event is 4, so n(E) = 4.
Step 3: Substitute the values of n(E) and n(S) in the probability formula, by 4 and 52, respectively. Then simplify the fraction.

The probability of drawing a five is 1/13.

Example 6: If you were to draw a card from a standard deck, what is the probability of NOT drawing a heart?

(1) $n(S) = 52$

(2) $n(E) = 39$

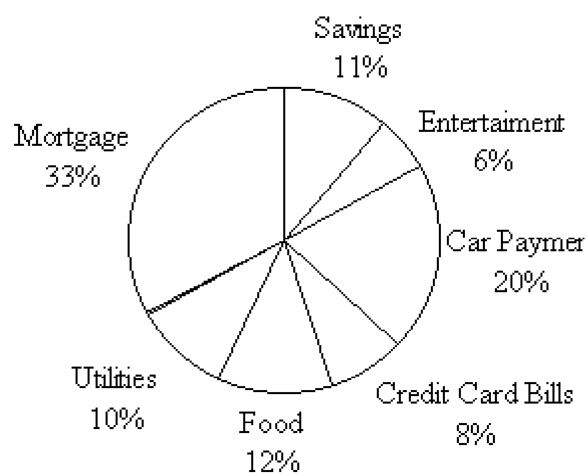
(3) $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{39}{52} = \dfrac{3}{4}$

Step 1: There are 52 cards in a standard deck, thus the number of elements in the sample space is 52, so n(S) = 52.
Step 2: There are four suits in a deck of cards. Therefore, three suits of cards are not hearts. Since each suit contains 13 cards, there are (3)(13) = 39 cards that are not hearts. Thus the number of elements in the event is 39, so n(E) = 39.
Step 3: Substitute the values of n(E) and n(S) in the probability formula, by 39 and 52, respectively. Then simplify the fraction.

The probability of not drawing a heart is 3/4.

Budget: Creation/Application
A budget is a plan for spending a fixed amount of money such as a monthly income for a family. A budget can be represented by a pie chart (circle graph). A pie chart is a graph in the form of a circle that is used to show how a whole is broken into parts. Each part of the circle graph represents a percent of the whole.

In the circle graph below, the Smiths use 33% of their monthly income to pay the mortgage, 20% is for their car payment, 12% of their monthly income is used to buy food, 11% is put into their savings account, 10% covers their utilities, 8% of their monthly income pays the credit card bills, and 6% of their monthly income goes toward entertainment.



It is possible to determine the amount of money the Smith's spend on each portion of their budget if we know the amount of their monthly income. It is also possible to determine the amount of the Smith's monthly income if we know the amount of money they spend on one portion of their budget.

Example 1: If the Smith family has a

$2,350.26 monthly income, how much money do they spend on food per month?

(1) $2,350.26 x 12% = money spent on food
(2) $2,350.26 x 0.12 = 282.0312
(3) $282.03

Step 1: To determine the amount of money spent on food, multiply the total monthly income ($2,350.26) by 12%.
Step 2: Before we can multiply $2,350.26 by 12%, we must convert 12% into a decimal. This involves moving the decimal point of the percent two places to the left.

$$12\% = 12.0\% = .12$$

After converting the percent into a decimal, multiply $2,350.26 by 0.12.
Step 3: This problem is asking for a dollar amount, so we must round 282.0312 to the nearest cent. To do this, we round to the hundredths place 282.0312. The 1 to the right of the hundredths place tells us to leave the 3 and drop the rest of the numbers after the 3. Therefore, 282.0312 is rounded to $282.03. (If the number to the right of the hundredths place had been 5 or greater, we would have rounded the 3 to 4 and gotten $282.04.)

Answer: The Smiths spend approximately $282.03 on food each month.

Example 2: If the Smith family spends $775.59 on their mortgage, what is their monthly income?

(1)

$$I \cdot (33\%) = 775.59$$

(2)

$$I \cdot (.33) = 775.59$$

(3)

$$\frac{I \cdot (.33)}{(.33)} = \frac{775.59}{(.33)}$$

(4)

$$I = 2350.272727$$

(5)

$$I = \$2350.27$$

Step 1: Let the variable "I" equal the Smith's monthly income. We know the Smiths spend $775.59 on their mortgage and that their mortgage is 33% of their monthly income. If we multiply their monthly income by 33%, we can determine their mortgage payment, so I x (33%) = 775.59. We can use this equation to determine the monthly income.
Step 2: Before we can continue, we must convert 33% into a decimal. 33% = 0.33 (See **Example 1**, Step 2 for instructions about converting a percent to a decimal).
Step 3: Divide both sides of the equation by 0.33. This will isolate the variable "I" (monthly income) on one side of the equal sign.
Step 4: Divide 775.59 by 0.33 to get 2350.272727.
Step 5: This problem is asking for a dollar amount, so we must round 2350.272727 to the nearest cent. 2350.272727 is rounded to $2,350.27 (the reason the amount is 1¢ more than the monthly income stated in Example 1 is because $775.59 is a rounded number).

Answer: The Smith's monthly income is approximately $2350.27.

Example 3: The Smith family has a monthly income of $2,350.26. They have 2 credit cards, each with a balance of $1,100.00. They make equal payments each month. How much are the Smiths paying toward each credit card each month? If their

credit card balances do not increase, how long will it take them to pay off their credit cards?

(1) $2,350.26 x 8% = amount toward credit card bills
(2) $2,350.26 x 0.08 = 188.0208
(3) 188.0208 ÷ 2 = 94.0104
(4) $94.01

Step 1: We must first determine the amount of money the Smiths spend on credit card bills, so multiply their monthly income ($2,350.26) by the percentage they budget for credit card bills (8%).
Step 2: Before we can continue, we must convert 8% into a decimal. 8% = 0.08. Now multiply $2,350.26 by 0.08 to get 188.0208.
Step 3: Since the Smiths are paying off two credit cards and making equal payments, we need to divide 188.0208 by 2.
Step 4: Round 94.0104 to the nearest cent to determine the amount of money the Smiths pay toward each credit card each month. 94.0104 rounds to $94.01.

This is a two-part question. We have determined the Smith's monthly payment to each credit card. Now we need to determine how long it will take the Smiths to pay off both credit cards.

(5) $1,100.00 ÷ $94.01 = number of months
(6) 11.70088288
(7) 12 months

Step 5: Each of the Smith's two credit cards have a balance of $1,100.00 and they are making equal payments to each credit card, so it will take them just as long to pay off both credit cards as it will take for them to pay off just one of the credit cards. We can now divide the balance of one credit card ($1,100.00) by the monthly payment the Smiths make ($94.01) to determine the number of months.
Step 6: $1,100 ÷ $94.01 = 11.70088288 = number of months
Step 7: Since it will take more than 11 months to pay off the credit card bills, we must round up to the next whole month.

Answer: The Smith family pays $94.01 toward each of their credit cards each month and it will take them 12 months to pay both credit cards off.

It is also possible to discuss a budget without the aid of a pie chart. Here is an example:

Example 4: Jimmy spends three times as much on his mortgage as he does on his car payment. His car payment is 18% of his monthly income of $1,926.31. He spends $90.00 a month on gasoline for his car, $230.00 a month on utility bills, and $219.35 on entertainment. (1) How much money does Jimmy spend on his house payment? (2) What percentage of his monthly income does Jimmy spend on entertainment?

Question #1:

(1) $1,926.31 x (18%) = car payment
(2) $1,926.31 x (0.18) = 346.7358
(3) 346.7358 x 3 = 1040.2074
(4) $1,040.21

Step 1: Since we know Jimmy's mortgage payment is 3 times his car payment, we can determine his car payment and multiply it by 3. To determine Jimmy's car payment we multiply his monthly income ($1,926.31) by 18%.
Step 2: Convert 18% into a decimal (18% = 0.18). Then multiply $1,926.31 by 0.18 to get 346.7358.
Step 3: Multiply 346.7358 (the car payment) by 3 to get 1040.2074 (the mortgage payment).
Step 4: Round 1040.2074 to the nearest cent. 1040.2074 rounds to $1,040.21.

Answer: Jimmy's mortgage payment is $1,040.21 each month.

Question #2:

(5) $219.35 ÷ $1,926.31 = 0.11387056

(6) 0.11387056 ~ 11.4% or 11%

Step 5: We know Jimmy spends $219.35 per month on entertainment. To determine what percent of his monthly income $219.35 is, we divide $219.35 by $1,926.31.
Step 6: We need to convert 0.11387056 into a percentage. This involves moving the decimal point two places to the right. Then round 11.387056%. Some teachers/tests may require rounding to the nearest tenth of a percent (11.4%) while others may require rounding to the nearest whole percent (11%).

Answer: Jimmy spends 11% of his monthly income on entertainment.

Inverse Variation

Variation equations are formulas that show how one quantity changes in relation to one or more other quantities. There are four types of variation: direct, inverse (or indirect), joint, and combined. This skill focuses on inverse variation.

Inverse (or indirect) variation formulas show that when one quantity increases, the other quantity decreases, and vice versa. For example, when the price of an item increases, the demand decreases. Indirect variation formulas are of the form $y = k/x$, where k is the constant of variation and needs to be determined.

Example 1: If p and q vary inversely and $p = 10$ when $q = 1.6$, find p when $q = 8$.

$$(1)\ p = \frac{k}{q} \to 10 = \frac{k}{1.6}$$

$$(2)\ 10(1.6) = \frac{k(1.6)}{1.6}$$
$$16 = k$$

$$(3)\ p = \frac{16}{8}$$
$$p = 2$$

Step 1: Set up the appropriate inverse variation formula. Since p varies inversely with q, that formula is $p = k/q$. Using the first set of values in the problem (because both p and q are known), substitute p and q into the formula so that the constant k can be determined.
Step 2: Solve for k by multiplying each side of the equation by 1.6 ($k = 16$).
Step 3: Now that the constant k has been determined, substitute the values into the formula that are known to determine the value of p when $q = 8$ and solve for p ($p = 2$).

Answer: $p = 2$ when $q = 8$

NOTE: A simpler way to solve inverse variation problems is to remember that the product of x and y is always the same. The values given for x and y may change, but the product of x and y remains the same. The following table is an example of a function in which x and y have an inverse relationship.

x	y	$x \times y$
2	4	8
-4	-2	8
-8	-1	8
1	8	8

As stated above, the values given for x and y change, but their products remain the same. For this table, the constant of variation, the product of x and y, is 8. Since y can always be found by dividing x into 8, y varies inversely with x.

Once the student is comfortable with the concept of inverse variation, he or she will be ready to solve problems in the context of

real world situations.

Example 2: The amount of time it takes a seamstress to sew a wedding dress varies inversely with the number of years of sewing experience she has. If Veronica could sew a wedding dress in 8 days when she had 3 years of sewing experience, how many days will it take her to sew a wedding dress now that she has 5.5 years of sewing experience? Round your answer to the nearest full day.

$$(1)\ \ 8 \times 3 = 24$$
$$(2)\ \ 24 \div 5.5 = 4.\overline{36}$$
$$(3)\ \ 4.\overline{36} \approx 4$$

Step 1: Determine the constant of variation by multiplying the two terms that vary inversely (8 days and 3 years).
Step 2: Divide the constant of variation by Veronica's experience now (5.5) to determine the number of days that it will take Veronica to sew the dress (4.3636...).
Step 3: Round 4.36 to the nearest whole day (4).

Answer: It will take Veronica 4 days to make a wedding dress now that she has 5.5 years of sewing experience.

As a reinforcement activity, have the student think of situations where two things vary inversely, such as the number of years in college versus the average college student's savings account. Make up some numbers for the various situations and solve for one variable when the other is known.

Equations: Systems
A system of equations contains at least two equations that may be linear, non-linear, or a combination of the two types. A graphical interpretation of the solution of a system of equations is that point (or points) where the graphs of the equations intersect. One method of finding the solution(s) of a system of equations involves adding the two equations together.

Example 1:

Starting Point
$3x + y = 13 \quad 2x - 4y = 18$

(1)
$$\begin{array}{r} 3x + \ \ y = 13 \\ +2x - 4y = 18 \\ \hline \end{array}$$

(2)
$$\begin{array}{r} 12x + 4y = 52 \\ +2x - 4y = 18 \\ \hline \end{array}$$

(3)
$$\begin{array}{r} 12x + 4y \\ +2x - 4y \\ \hline 14x + 0y \end{array}$$

(5)
$$\begin{array}{r} 2(5) - 4y = 18 \\ 10 - 4y = 18 \end{array}$$

(6)
$$\begin{array}{r} 10 - 4y = 18 \\ -10 \qquad -10 \\ \hline -4y = 8 \end{array}$$

(7)
$$\frac{-4y}{-4} = \qquad y =$$

Check Your Work
$$3(5) + (-2) = 13 \qquad 2(5) - 4(-2)$$
$$15 - 2 = 13 \qquad 10 + 8$$
$$13 = 13 \qquad 18$$

Step 1: Write the equations in a vertical format, aligning the x-terms, y-terms, equal signs, and constant terms.
Step 2: The objective is to add the corresponding parts of the two equations together and eliminate either the x- or y-term. Multiplying each term in the top equation by 4 will create the necessary conditions for eliminating the y-term.
Step 3: Add like terms in the two equations, and notice the resulting y-term will have a coefficient of zero and be eliminated.
Step 4: Solve the resulting equation for x. In this case, that means divide both sides of the equation by 14. This results with x = 5.
Step 5: To solve for y, substitute the value of x (5) in either of the original equations. The second equation was chosen for this example.
Step 6: Subtract 10 from both sides of the equation.
Step 7: Solve the resulting equation for y. In this case, that means divide both sides of the equation by -4. This results with y = -2.

As with all other equations, substitute the values of x and y into the original equations to ensure they are correct solutions.

Both values check out. Therefore, the

solution to the system of equations are x = 5 and y = -2. This can be interpreted as the ordered pair (5, -2) of the point of intersection of the graphs of these two equations.

Equations of a Line
Every line on any coordinate graph has a corresponding equation which describes every point on the line. Every linear equation (equation of the line) contains a slope. The slope of a line is the same between any two points on the line.

Before you can find the equation of a line, you must first be able to find the slope of a line when given two coordinate points on the line. These two points are named: (x_1, y_1) and (x_2, y_2). The formula for the slope of a line follows.

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Find the slope of the line between Point R (2, 4) and Point S (1, 3).

(1) $$m = \frac{3-4}{1-2}$$

(2) $$m = \frac{-1}{-1}$$ $$m = 1$$

Step 1: Substitute the given coordinate points into the formula.
Step 2: Simplify the fraction.

Answer: The slope of the line is 1.

The Point-Slope form for the equation of a line:

$$y - y_1 = m(x - x_1)$$

Example 2: Use the following points to find the equation of the line.
Point T (7, -3)
Point U (-4, 6)

(1)
$$m = \frac{6 - -3}{-4 - 7}$$
$$m = \frac{6+3}{-4-7}$$
$$m = \frac{9}{-11} = -\frac{9}{11}$$

(2)
$$y - -3 = -\frac{9}{11}(x - 7) \quad y + $$

Step 1: Solve for the slope of the line between Point T and Point U.
Step 2: Use one of the coordinate points and the slope and substitute them into the Point-Slope form for the equation of a line.
Step 3: Simplify both sides of the equation.
Step 4: Subtract 3 from both sides of the equation.

The equation of the line that passes through (7, -3) and (-4, 6) is y = -9/11x + 30/11.

Number Relation Problems
Number relation problems involve sentences that must be translated into equations.

To translate sentences, review the following phrase/number equivalences:

If x is equal to 4, then x = 4
If the sum of 2x and 2y is 23, then 2x + 2y = 23
If 3 is less than twice y, then 2y > 3
If 5x decreased by y is more than 2y, then 5x - y > 2y
If the result of 7 more than 3 times x is y, then 3x + 7 = y

Example: One number is 2 less than another number. If twice the larger number is decreased by 3 times the smaller number, the result is 20.

Set Up Equations

Step 1: $x + 2 = y$

Step 2: $2y - 3x = 20$

Solve for "x"

Step 3: $2(x + 2) - 3x = 20$

Step 4: $2x + 4 - 3x = 20$

Step 5: $-x + 4 = 20$

Step 6: $-x = 16$

Step 7: $x = -16$

Solve for "y"

Step 8: $2y - 3(-16) = 20$

Step 9: $2y + 48 = 20$

Step 10: $2y = -28$

Step 11: $y = -14$

Check Your Work

$x + 2 = y$	$2y - 3x = 20$
$(-16) + 2 = -14$	$2(-14) - 3(-16) = 20$
$-14 = -14$	$-28 - -48 = 20$
	$20 = 20$

Step 1: Translate the first sentence in the problem into the first equation.
Let x and y represent the unknown numbers, with y representing the larger number and x representing the smaller number.
Step 2: Translate the second sentence in the problem into the second equation.
Use x and y as in the first equation, substituting them into their respective locations in the equation.
Step 3: Substitute x + 2 for y in the second equation.
Step 4: Distribute the 2 across (x + 2).
Step 5: Combine the similar terms.
Step 6: Subtract 4 from both sides.
Step 7: Divide each side by -1 to find the value of x.
Step 8: Using x = -16, substitute for x in the second equation.
Step 9: Multiply -3 and -16.
Step 10: Subtract 48 from both sides.
Step 11: Divide each side by 2.

Answer: x = -16 and y = -14

As always, check the solutions by substituting both equations with x = -16 and y = -14.

Factoring

Consider the following equation:

$3 \times 4 = 12$

The numbers 3 and 4 are said to be factors of the number 12. This concept of factoring is not reserved for numbers, but may be extended to polynomials as well.

Factoring is the breaking up of quantities into products of their component factors. One way to think of factoring is as the opposite or inverse of multiplying.

A polynomial is a term or sum of terms. Each term is either a number or a product of a number and one or more variables.

A monomial is a polynomial with one term.

A binomial is a polynomial with two terms.

A trinomial is a polynomial with three terms.

Consider the following polynomial:

$$4y^3 + 16y^2 - 20y$$

A typical question on factoring will include a polynomial like the one above. Notice that 4y is a common factor of each term of the polynomial.

Step 1: Factor out the 4y by dividing each term of the trinomial by 4y.

$$\frac{4y^3}{4y} + \frac{16y^2}{4y} - \frac{20y}{4y}$$

$$4y\left(y^2 + 4y - 5\right)$$

Step 2: The trinomial in parentheses can be factored further. Since the coefficient of the "y squared" term is equal to 1, focus on the last term, in this case, -5. If factors of -5 can be found that ADD up to the coefficient of the middle term (in this case, 4) the trinomial can be factored. Two factors of -5 are
5 and -1, and when ADDED together the result is equal to the coefficient of the middle term, 4. Notice how these numbers are put together to construct the fully

factored trinomial:

(4y)(y + 5)(y - 1)

To check the result, use the rules for multiplying polynomials and you should have the original polynomial when finished.

Sometimes factoring must be done by grouping. The polynomial given below may appear impossible to factor at first, but if you examine the steps you will see a method to use with polynomials of this type.

$$15x^2 + 20xy + 18nx + 24ny$$

$$\text{Step 1}: 5x(3x + 4y) + 18nx + 24ny$$

$$\text{Step 2}: 5x(3x + 4y) + 6n(3x + 4y)$$

$$\text{Step 3}: (5x + 6n)(3x + 4y)$$

Step 1: (5x) is a common term to both 15x and 20xy. Factor it out of only those two terms.
Step 2: (6n) is a common term of both 18nx and 24ny. Factor it out of only those two terms.
Step 3: Notice that the quantity (3x + 4y) is a common factor of 5x and 6n. The expression is rewritten to indicate this, and the polynomial is completely factored.

A special type of polynomial expresses the difference of two perfect squares. Polynomials of this type are factored easily once the rule is remembered.

$$x^2 - 36$$

$$(x + 6)(x - 6)$$

Since each term in the polynomial is a perfect square, the square root of each term (in this case x and 6 respectively) will be used in the following way. The original polynomial is factored as (x + 6)(x - 6). Notice that if these terms are multiplied together, the original polynomial is formed. Polynomials that are in the "difference of squares" form may always be factored as the sum of the square roots times the difference of the square roots.

Quadratic Formula
A quadratic equation is a polynomial equation in which the highest power of the unknown variable is two.

An example of a quadratic equation is below.

$$x^2 + 6x - 91 = 0.$$

The format of a quadratic equation is $ax^2 + bx + c = 0$. Quadratic equations can be solved by factoring, graphing, or by using the quadratic formula. The quadratic formula is as follows:

$$\boxed{\begin{array}{l}\text{Quadratic Formula}\\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}\end{array}}$$

It can be found in any algebra textbook. This formula should be memorized.

To apply the formula to a quadratic equation, use the quadratic equation format given above as a guideline.

Example 1: Solve the quadratic equation.

$$x^2 + 6x - 91 = 0$$

(1)

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-91)}}{2(1)}$$

(2)

$$x = \frac{-6 \pm \sqrt{400}}{2}$$

(4)

$$x = \frac{-6 + 20}{2} \text{ and } x = \frac{-6 - 20}{2}$$

$$x = \frac{14}{2} \qquad x = \frac{-26}{2}$$

$$x = 7 \text{ and } x = -13$$

Step 1: Determine the values of a, b, and c and substitute them into the quadratic formula. a = 1, b = 6, and c = -91

Step 2: Determine the value under the radical symbol. 6 squared is 36 and -91 times -4 equals 364.
36 + 364 = 400
Step 3: The square root of 400 is 20 (20 x 20 = 400).
Step 4: Split the remaining problem into two problems: (-6 + 20) ÷ 2 and (-6 - 20) ÷ 2 and solve the two problems.

The answers are x = 7 and x = -13.

Example 2: Solve the quadratic equation.

$$5x^2 + 2x + 8 = 4x^2 - 2x + 4$$

(1) $5x^2 + 2x + 8 = 4x^2 - 2x + 4$

(2)
$$\begin{array}{r}
5x^2 + 2x + 8 = 4x^2 - 2x + 4 \\
-4x^2 \qquad\qquad\quad -4x^2 \qquad\qquad \\
\hline
x^2 + 2x + 8 = -2x + 4 \\
+2x \qquad\quad +2x \qquad \\
\hline
x^2 + 4x + 8 = \;\; 4 \\
-4 \;\; -4 \\
\hline
x^2 + 4x + 4 = 0
\end{array}$$

(3) $a = 1,\ b = 4,\ c = 4$

(4) $x = \dfrac{-4 \pm \sqrt{(4)^2 - 4(1)(4)}}{2(1)}$

(5) $x = \dfrac{-4 \pm \sqrt{16 - 16}}{2} = \dfrac{-4 \pm 0}{2}$

(6) $x = \dfrac{-4}{2} = -2$

Step 1: Write the equation.
Step 2:
Subtract $4x^2$ from both sides of the equation.
Then, add 2x to both sides of the equation.
Finally, subtract 4 from both sides of the equat

This will put the equation in standard form.
Step 3: Determine the values of a, b, and c.
Step 4: Substitute the values of a, b, and c into the quadratic formula.
Step 5: Determine the value under the

radical sign. The square root of 0 is 0.
Step 6: Solve for x.

Answer: x = -2

The discriminant is the portion of the quadratic equation under the radical sign $b^2 - 4ac$. The discriminant properties below will give you vital information about quadratic equations.

1. If the discriminant is a perfect square, then the quadratic equation can be factored.
2. If the discriminant is greater than 0, then the equation has two real solutions.
3. If the discriminant is less than 0, then the equation has no real solutions.
4. If the discriminant is equal to 0, then the equation has one real solution.

Example 3: How many solutions does the following quadratic equation have?

$$3x^2 + 5x - 12 = 0$$
$$(1)\ \ a = 3,\ b = 5,\ c = -12$$
$$(2)\ \ (5)^2 - 4(3)(-12)$$
$$(3)\ \ 25 + 144 = 169$$

Step 1: Determine the values of a, b, and c.
Step 2: Substitute the values for a, b, and c into $b^2 - 4ac$. Step 3: Simplify the discriminant.

Since the discriminant is greater than zero, there are two real solutions.

If the discriminant is a perfect square, then the solutions are rational.
If the discriminant is not a perfect square, then the solutions are irrational.

Polynomials: Addition

A monomial is the product of a number and an unknown variable or unknown variables. 6xy is a monomial. The sum of two or more monomials is called a polynomial. Here is an example of a polynomial:
$y^2 + 4y + 3$.

Adding and subtracting polynomials includes simplifying and combining "like" terms. Like terms are monomials that have the same variable or variables for which the variable or variables have the same exponent.

Examples:

$$\begin{Bmatrix} 2x \\ 4x \end{Bmatrix} \text{like terms} \qquad \begin{Bmatrix} 2x \\ -4x^2 \end{Bmatrix} \text{not like terms}$$

To add polynomials, combine similar terms.

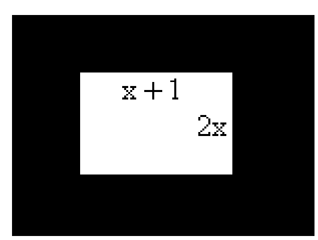
Example 1: $(p^2 + 3p + 3) + (p^2 - 2p - 6)$

(1)	**(2)**
$p^2 + 3p + 3 + p^2 - 2p - 6$	$p^2 + p^2 = 2p^2$
	$3p - 2p = p$
	$3 - 6 = -3$

Step 1: Set the two polynomials up as one long polynomial.
Step 2: Combine the like terms.
Step 3: Add the results of combining the like terms to determine the answer.

The sum is $2p^2 + p - 3$.

Example 2:



$$(1)\ \ 3x^2 + 4x - 2 + (2x^2 + 2x)$$
$$(2)\ \ (3x^2 + 2x^2) + (4x + 2x) - 2$$
$$(3)\ \ 5x^2 + 6x - 2$$

Step 1: The area of the large rectangle would be the area of the shaded region added to the

area of the small rectangle. Since we know both areas, we simply add them together.
Step 2: Collect like terms so they can be added.
Step 3: Add together any like terms that were collected to determine the final answer.

The area of the large rectangle is
$5x^2 + 6x - 2$.
Sometimes it is necessary to use the distributive property before we can combine like terms.

Example 3:
$3\left(8x^2 + 16x - 7\right) - 4\left(x^2 + 3x - 5\right)$

(1) $3(8x^2) + 3(16x) + 3(-7) + (-4)(x^2) + (-4)(3x) + (-4)(-5)$
$24x^2 + 48x - 21 - 4x^2 - 12x + 20$
(2) $(24x^2 - 4x^2) + (48x - 12x) + (-21 + 20)$
(3) $20x^2 + 36x - 1$

Step 1: Multiply each term of the first polynomial by 3. Then multiply each term of the second polynomial by -4.
Step 2: Group like terms together.
Step 3: Combine like terms.

Answer: $20x^2 + 36x - 1$
Example 4: Solve for a, b, and c.

$\left(9x^2 + bx + 4\right) + \left(ax^2 - 5x - 3\right) = 5x^2 - 7x + c$

(1) $(9x^2 + ax^2) + (bx - 5x) + (4 - 3) = 5x^2 - 7x$
(2) $9x^2 + ax^2 = 5x^2$
$bx - 5x = -7x$
$4 - 3 = c$
(3) $a = -4, b = -2, c = 1$

Step 1: Group the similar terms on the left side of the equation together.
Step 2: Now, group like terms from both sides of the equal sign together.
Step 3: Solve for a, b, and c.

Answer: a = -4, b = -2, and c = 1

Exponential Notation - E
Exponents communicate the number of times a base number is used as a factor. The base number 5 to the 3rd power (with an exponent of 3) translates to 5 x 5 x 5. (5 to the 3rd power is not 5 x 3.) The result of 5 to the 3rd power is 125. To perform operations with exponents, all exponential properties must be understood.

To simplify an expression or to find the missing term in a simplified expression, apply the following exponential properties which are listed below.

Product of Powers:

When multiplying two (or more) numbers

with the same base that have exponents, the base remains the same and the exponents are added.

$$a^m \times a^n = a^{m+n}$$
$$5^3 \times 5^6 = 5^{3+6} = 5^9$$

Power to a Power:

When taking a number with an exponent to another power, the base remains the same and the exponents are multiplied.

$$(a^m)^n = a^{m \cdot n}$$
$$(5^3)^6 = 5^{3 \cdot 6} = 5^{18}$$

The Power of Zero:

Any number taken to the power of zero (except zero) equals 1.

$$a^0 = 1$$
$$5^0 = 1$$
$$100^0 = 1$$

Negative Exponents:

There is a rule for evaluating negative exponents.

$$a^{-m} = \frac{1}{a^m}$$
$$4^{-3} = \frac{1}{4^3}$$

Example 1: Simplify. $(3xz^3)(2x^4z^2)$

$$\overset{(1)}{(3 \cdot 2)(x^1 \cdot x^4)(z^3 \cdot z^2)} \qquad \overset{(2)}{(6)(x^{1+4})(z^{3+2})} \qquad \overset{(3)}{6x^5z^5}$$

Step 1: Separate the expression into products of whole numbers times products of variables with like bases. A variable or number that is written without an exponent automatically has an exponent of 1.
Step 2: First, multiply the whole numbers to get 6. Use the "Product of Powers" rule to evaluate the terms in the second set of parentheses and the terms in the third set of parentheses.
Step 3: Add the exponents to complete the expression.

Answer: $6x^5z^5$

Example 2: Simplify. $(-2a^3b)^2$

$$\overset{(1)}{(-2)^2 \cdot (a^3)^2 \cdot (b)^2} \qquad \overset{(2)}{(-2)(-2) \cdot (a^{3 \cdot 2}) \cdot (b^{1 \cdot 2})}$$

Step 1: Separate the expression into products of each of its terms. The exponent (2) is given to each term inside the parentheses.
Step 2: -2 to the second power is (-2)(-2). Use the "Power to a Power" rule to evaluate the rest of the expression.
Step 3: Multiply the powers to determine the exponents on the variables. The final expression can be written with the parentheses still in it, or without the parentheses. It is more acceptable to remove the parentheses.

Polynomials: Multiplication
A monomial is the product of a number and an unknown variable or unknown variables. 6xy is a monomial. The sum of two or more monomials is called a polynomial. Here is an example of a polynomial:
$y^2 + 4y + 3$.
A binomial is a polynomial with exactly two

monomial terms. 3x + 4 is a binomial. A <u>trinomial</u> is a polynomial with exactly three terms. 4xy - 3x +6y is a trinomial.

Adding and subtracting polynomials includes simplifying and combining "like" terms. <u>Like terms</u> are monomials that have the same variable or variables for which the variable or variables have the same exponent.

Examples :

$\left\{\begin{matrix} 2x \\ 4x \end{matrix}\right\}$ like terms $\qquad$ $\left\{\begin{matrix} 2x \\ -4x^2 \end{matrix}\right\}$ not like terms

To multiply monomials and polynomials, the exponential properties must be followed. These properties apply to all real numbers with positive exponents.

Exponential Properties for Multiplication
$(a^m)(a^n) = a^{m+n}$
$(a^m)^n = a^{(m)(n)}$
$(ab)^m = (a^m)(b^m)$

Exponential property #1 can be used to obtain results for problems such as:

$$(p^3r^3)\ (p^2r)\ (p^2r^3) = p^{3+2+2}r^{3+1+3} = p^7r^7$$

Exponential properties #2 and #3 can be used to obtain results such as:

$$(3p^2r^3)^3 = (3^3)\ (p^{(2)(3)})\ (r^{(3)(3)}) = 27p^6r^9$$

To multiply a monomial and a polynomial, multiply the monomial by each term of the polynomial. In other words, distribute the monomial.

$$3x(x^2 + 3x - 4)$$
$$(3x)(x^2) + (3x)(3x) + (3x)(-4)$$
$$3x^3 + 9x^2 - 12x$$

To multiply two polynomials, each term of the first polynomial must be multiplied by each term of the second polynomial. Distribute each term of the first polynomial across the second polynomial.

$$(3x - 4)(4x - 2)$$
$$(3x)(4x) + (3x)(-2) + (-4)(4x) + (-4)(-2)$$
$$12x^2 - 6x - 16x + 8$$
$$12x^2 - 22x + 8$$

Example 1: Which choice below represents the following trinomial:

$2n^2 + 7n + 6$

A. (3n + 2) (n + 2)
B. (2n + 3) (n + 2)
C. (2n - 3) (n - 2)

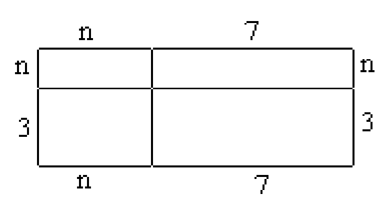
We know that when multiplying two polynomials, each term of the first polynomial must be multiplied by each term of the second polynomial. Let's look at our solutions.

A. $(3n + 2)(n + 2) = 3n^2$ ← we can eliminate

B. $(2n + 3)\ (n + 2) = 2n^2 + 3n + 4n + 6 =$
$2n^2 + 7n + 6$

Answer: B

Example 2: Given these measurements, find the area of the rectangle.



(1)	(2)	(3)	(4)
Length $= n + 7$ Width $= n + 3$	$(n+7)(n+3)$	$(n+7)(n+3)$ $n \cdot n + n \cdot 3 + 7 \cdot n + 7 \cdot 3$ $n^2 + 3n + 7n + 21$	$n^2 + 3n + 7n + 21$ $n^2 + 10n + 21$

Step 1: Area = Length x Width. Determine the length and width of the rectangle.
Step 2: Multiply the length and width to determine the area of the rectangle.
Step 3: Use FOIL to multiply the two binomials. Remember, multiply the **first** terms in each binomial, then multiply the **outer** terms, next multiply the **inner** terms, and finally multiply the **last** terms.
Step 4: The final step in solving this problem is to add the like terms. 3n + 7n = 10n

Answer: $n^2 + 10n + 21$

Polynomials: Division
A monomial is the product of a number and an unknown variable or unknown variables. 6xy is a monomial. The sum of two or more monomials is called a polynomial. Here is an example of a polynomial:
$y^2 + 4y + 3$.
A binomial is a polynomial with exactly two monomial terms. 3x + 4 is a binomial. A trinomial is a polynomial with exactly three terms. 4xy - 3x +6y is a trinomial.

Before dividing polynomials, recall the following properties associated with exponents:

Exponential Properties for Division

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^0 = 1$$

Example 1: Divide.

$$\frac{12x^3y}{-3xy}$$

(1)	(2)	(3)
$\frac{12}{-3} = -4$	$\frac{x^3}{x} = x^{3-1} = x^2$	$\frac{y}{y} = y^{1-1} = y^0 =$

Step 1: Divide the whole numbers: 12 ÷ -3 = -4.
Step 2: Use the properties above to divide the variables. Begin with the x-variables. x-cubed divided by x equals x-squared.
Step 3: Now divide the y-variables. y divided by y equals y to the power of zero. Any number taken to the power of zero equals 1.
Step 4: Finally, multiply the quotients back together.

The answer is $-4x^2$.

Dividing a Polynomial by a Monomial:

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial. Then, combine the similar terms.

Example 2: Divide.

$$\frac{3m - 9n}{3}$$

$$\textbf{(1)}\quad \frac{3m}{3} = m \qquad \textbf{(2)}\quad \frac{-9n}{3} = -3n \qquad \textbf{(3)}\quad m - 3n$$

Step 1: Divide 3m by 3, to get m.
Step 2: Divide -9n by 3, to get -3n.
Step 3: Combine the terms.

Answer: m - 3n

Dividing a Polynomial by a Polynomial:

Dividing one polynomial by another is very similar to long division.

Example 3: Divide $(6x^2 + 8x + 8)$ by $(3x + 1)$.

$$\begin{array}{ll}
\text{Step 1:} & 3x+1\overline{\big)\,6x^2+8x+8} \\
 & \qquad\qquad\quad 2x+2 \\
\text{Step 2:} & 3x+1\overline{\big)\,6x^2+8x+8} \\
\text{Step 3:} & \quad -\underline{(6x^2+2x)} \\
\text{Step 4:} & \qquad\qquad 6x+8 \\
\text{Step 5:} & \qquad\quad \underline{-(6x+2)} \\
 & \qquad\qquad\quad 6
\end{array}$$

Step 1: Write the problem as a long division problem. The binomial belongs on the outside of the division symbol because it is the term we are dividing by.
Step 2: Now, we can begin dividing. $(3x)(2x) = 6x^2$ So, 2x belongs above the 8x.
Step 3: The next step is to multiply 2x by (3x + 1).

$(2x)(3x+1) = 6x^2 + 2x$ Subtract that product from $6x^2 + 8x$. Now, bring the + 8 straight down beside the 6x.
Step 4: (3x)(2) = 6x, so we place the 2 above the 8 in the answer.
Step 5: Multiply 2 by (3x + 1) to get 6x + 2. Subtract (6x + 2) from (6x + 8). There is a remainder of 6, so we write the remainder as a fraction with the binomial as the denominator.

Answer: $2x + 2 + \frac{6}{3x+1}$

Expressions: Evaluating & Simplifying
Expressions look like equations except expressions do not have equal (=) signs. Expressions are "evaluated" or "simplified," not "solved."

To evaluate and simplify expressions with brackets and parentheses, use the rules regarding Order of Operations, Integer Properties, and Evaluating.
Here is the order of operations:

(1) Parentheses, Brackets, and Braces
(2) Exponents or Roots
(3) Multiply or Divide in order from left to right
(4) Add or Subtract in order from left to right

The order of operations is the same whether you are working with whole numbers, fractions, or decimals. Here are a few helpful hints for using the order of operations. First, remember to complete all operations of one type before moving on to the next type (for example, complete all multiplication and division before moving on to addition or subtraction). Second, remember that when working the multiplication or division move from the left to the right (for example, 2 x 6 ÷ 3. In this case, you would multiply first because the multiplication is the first operation when reading from the left to the right). Finally,

addition and subtraction work the same way as multiplication and division - from the left to the right (for example, 10 - 6 + 2. In this case, you would subtract 6 from 10 first, then add the 2).

Evaluating an expression can occur in two ways. If the expression contains only operations and numbers, simply perform the indicated operations, observing the correct order. If the expression contains variables as well, number values will have to be substituted for each variable, and then simplified using the correct order of operations.

Example 1: Evaluate 3x + 2 if x = -2

(1) 3(-2) + 2
(2) -6 + 2
(3) -4

Step 1: Substitute -2 into the expression in place of the x's.
Step 2: Use the order of operations and multiply 3 and -2 first.
Step 3: Complete the addition.

Some expressions may have two variables. If values are given for both variables, substitute each value into their respective variable within the given expression.

Example 2: Evaluate 3(x - 3y) if x = -2 and y = 2

(1) 3(-2 - 3(2))
(2) 3(-2 -6)
(3) 3(-8)
(4) -24

Step 1: Substitute the values of x and y into the expression.
Step 2: Perform the multiplication that is within parentheses first.
Step 3: Subtract the numbers inside the parentheses.
Step 4: Multiply to complete the problem.

Example 3:

Evaluate [3+2(-4y + 2x)] - 5x if x = -1 and y

Step 1 [3 + 2(-4 + -2)] + 5
Step 2 [3 + 2(-6)] + 5
Step 3 [3 - 12] + 5
Step 4 [-9] + 5

Step 1: Substitute the values given for x and y wherever x and y are in the expression.
Step 2: Simplify within the parentheses.
Step 3: Multiply within the brackets.
Step 4: Add within the brackets.

Answer: -4

Radicals: Equations

Radical equations are equations in which there are numbers and or letters inside the radical sign. The numbers and/or letters are called the radicand. The following is an example of how to solve a radical equation.

Example 1: Find the solution set of the radical equation.

$$4+\sqrt{4x-3}=7$$

(1)

$$\begin{array}{rcl} 4+\sqrt{4x-3} & = & 7 \\ -4 & & -4 \\ \hline \sqrt{4x-3} & = & 3 \end{array}$$

(2)

$$\left(\sqrt{4x-3}\right)^2 = 3^2$$

(3)

$$4x-3$$

(4)
$$\begin{array}{r}4x-3=9\\ \underline{\quad +3\;\; +3}\\ 4x\qquad =12\end{array}$$

(5)
$$\frac{4x}{4}=\frac{12}{4}$$
$$x=3$$

Step 1: Combine the whole numbers by subtracting 4 from both sides.
Step 2: Square both sides.
Step 3: Simplify each side of the equation.
Step 4: Add 3 to both sides of the equation.
Step 5: Divide both sides of the equation by 4.

The answer is: x = 3.
Substitute the result into the original equation to verify that it is correct.

Example 2: The square root of the quantity of 3 times a number increased by 4, equals 11. Find the number.

(1) $\sqrt{3x+4}=11$

(2) $\left(\sqrt{3x+4}\right)^2=(11)^2$

(3) $3x+4=121$

(4)
$$\begin{array}{r}3x+4=121\\ \underline{\quad -4\quad -4}\\ 3x\qquad =117\end{array}$$

(5) $\frac{3x}{3}=\frac{117}{3}$
$$x=39$$

Step 1: Write the equation.
Step 2: Square both sides of the equation to get rid of the square root.
Step 3: Simplify both sides of the equation.
Step 4: Subtract 4 from both sides of the equation.
Step 5: Divide both sides of the equation by 3.

The answer is: x = 39.

Sets/Subsets/Solution Sets

A set is a collection of numbers. A subset is a collection of numbers that all belong to a larger set. Every set is also a subset of itself. If Set C is a subset of Set D, then every element of Set C is also in Set D. Set C is a part of (a subset of) Set D. A solution set of an open sentence is the set of all elements which make the sentence true when replaced for the unknown.

Sets are indicated by braces,
$\{\quad\}$.
The empty set, $\{\varnothing\}$, is a subset of every set.

Example 1: Find the solution.

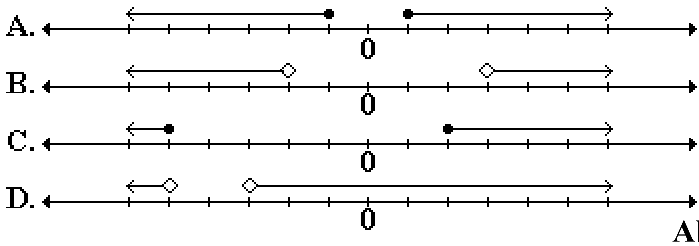
2x + 3 = 3(x + 3)

$$\begin{array}{lr}
 & \mathbf{2x+3=3(x+3)}\\
\mathbf{(1)} & \mathbf{2x+3=3(x)+3(3)}\\
 & \mathbf{2x+3=3x+9}\\
\mathbf{(2)} & \underline{\mathbf{-2x\qquad -2x}}\\
 & \mathbf{3=1x+9}\\
\mathbf{(3)} & \underline{\mathbf{-9\qquad -9}}\\
 & \mathbf{-6=1x}\\
\mathbf{(4)} & \mathbf{x=-6}
\end{array}$$

Step 1: Distribute the 3 to the numbers in the parentheses. This involves multiplying each term in the parentheses by 3.
Step 2: We want the variable on one side by itself. Subtract 2x from each side of the equation to get the variable on one side of the equation.
Step 3: Subtract 9 from each side of the equation to isolate the variable on one side of the equation.
Step 4: Rewrite -6 = 1x with the variable first. x = -6

Answer: x = -6

Example 2: Choose the corresponding number line: y < -2 or y > 3.

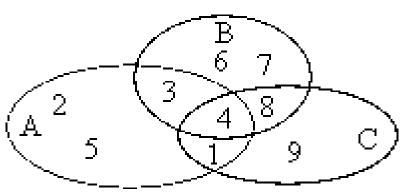


The answer is B.

We are also able to determine sets and subsets from Venn diagrams. Venn diagrams are drawings that show relationships among sets of items. Venn diagrams are usually sets of overlapping circles or ovals. The relationships between the sets can be represented by the following symbols.

Symbol	Word Meaning	Example	Definition
$\cup$	Union	$A \cup B$	The terms that A and B do NOT have in common.
$\cap$	Intersection	$A \cap B$	The terms that A and B have in common.

Example 3: Given the sets A, B, and C expressed in the diagram, which option is NOT true?



A. $A \cap C = \{1, 4\}$
B. $A \cup C = \{2, 3, 5, 8, 9\}$
C. $B \cap C = \{4, 8\}$
D. $B \cup C = \{3, 6, 7, 9\}$

Step 1: Determine the numbers that are in each set.

Set $A = \{1, 2, 3, 4, 5\}$
Set $B = \{3, 4, 6, 7, 8\}$
Set $C = \{1, 4, 8, 9\}$

Step 2: Use the sets to determine which of the four answer choices is NOT true.

Answer: Choice D is NOT true.

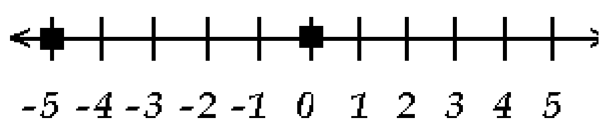
Absolute Value: Solve

The absolute value of a real number is the distance the real number x is from 0 on a number line. The absolute value of a real number is denoted by placing the real number within two vertical lines: |x|. In other words, |-5| ("the absolute value of -5") is 5 because -5 is 5 units from 0 on a number line.

There are two major principles of absolute value:

1. The absolute value of a negative number is always a positive number.
2. The absolute value of 0 is 0.

One way to find the absolute value of single digit numbers such as |-5|, is to draw a number line and plot -5 and 0 on the number line.



Notice that -5 is 5 points from 0, so the absolute value of |-5| is 5.

The student must also be able to find the absolute value of single digit numbers, solve absolute value equations, evaluate absolute value expressions, and recognize absolute value expressions on number lines.

The following example demonstrates how to solve equations with absolute value.

Example 1: Solve: |2x - 3| = 17

(1)

$$2x - 3 = 17 \text{ and } 2x - 3 = -17$$

(2)

$$\begin{array}{r} 2x - 3 = 17 \\ +3 \quad +3 \\ \hline \frac{2x}{2} = \frac{20}{2} \\ x = 10 \end{array}$$

(3)

$$\begin{array}{r} 2x - 3 = -17 \\ +3 \quad +3 \\ \hline \frac{2x}{2} = \frac{-14}{2} \\ x = -7 \end{array}$$

<u>Step 1:</u> Set the equation up to equal both 17 and -17. This is because 2x - 3 can equal either 17 or -17 to make the original equation true.
<u>Step 2</u>: Solve the equation 2x - 3 = 17. Add 3 to both sides of the equation to get 2x = 20. Then divide each side of the equation by 2 to get x = 10.
<u>Step 3</u>: Solve the equation 2x - 3 = -17. Add 3 to both sides of the equation to get 2x = -14. Then divide each side of the equation by 2 to get x = -7.

The solutions for |2x - 3| = 17 are **x = 10 or x = -7**. However, to be sure, always check solutions by substituting both solutions (10 and -7) for x (in |2x - 3| = 17). For x = 10 (shown in Step 4) and for x = -7 (shown in Step 5):

(4)

$$\begin{aligned} 2(10) - 3 &= 17 \\ 20 - 3 &= 17 \\ 17 &= 17 \end{aligned}$$

(5)

$$\begin{aligned} 2(-7) - 3 &= -17 \\ -14 - 3 &= -17 \\ -17 &= -17 \end{aligned}$$

Example 2: Evaluate expression |x - 3| when x = -2.

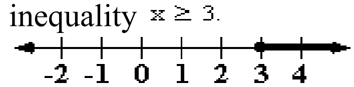
(1) |-2 - 3|
(2) |-5| = 5

<u>Step 1</u>: Substitute -2 in the absolute value expression for x.
<u>Step 2</u>: The expression |-2 - 3| becomes |-5|. The absolute value of |-5| is 5 because -5 is 5 points from 0 on a number line.

It may be useful to review the inequality symbols.

Symbol	**Definition**	**Type on nu**
$<$	**is less than**	**o**
$>$	**is greater than**	**o**
$=$	**is equal to**	**cl**
$\leq$	**is less than or equal to**	**cl**
$\geq$	**is greater than or equal to**	**cl**
$\neq$	**is not equal to**	**o**

Inequalities can be represented as a value on a number line. The following number line represents the inequality $x \geq 3$.



Example 3: Solve and graph the inequality.
|3x + 5| < 10

(1)

$$3x + 5 < 10 \quad \text{and} \quad 3x + 5 > -10$$

(2)

$$\begin{array}{r} 3x + 5 < 10 \\ -5 \quad -5 \\ \hline \dfrac{3x}{3} < \dfrac{5}{3} \\ x < \dfrac{5}{3} \end{array}$$

(3)

$$\begin{array}{r} 3x + 5 > -10 \\ -5 \quad -5 \\ \hline \dfrac{3x}{3} > \dfrac{-15}{3} \\ x > -5 \end{array}$$

Step 1: Set up the inequality as being less than 10 and greater than -10. (The second inequality should have the "is greater than" symbol because: when we make the 10 negative, we turn the inequality symbol the opposite direction. In this case, we change it from "is less than" to "is greater than.")
Step 2: Solve the inequality 3x + 5 < 10. Subtract 5 from both sides of the inequality, then divide both sides of the inequality by 3.
Step 3: Solve the inequality 3x + 5 > -10. Subtract 5 from both sides of the inequality, then divide both sides of the inequality by 3.

The answer is: x < 5/3 and x > -5, which can also be written as -5 < x < 5/3.

Now, graph the inequality. Place an open dot on the -5 and an open dot on 5/3 because the inequality is strictly less than 5/3 and strictly greater than -5 (x cannot equal 5/3 or -5). Shade the portion of the graph that is greater than -5 and less than 5/3.